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USE OF ENTIRE EIGENSTRUCTURE ASSIGNMENT WITH HIGH-GAIN ERROR-ACTUATED FLIGHT CONTROL SYSTEMS

THESIS

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USE OF ENTIRE EIGENSTRUCTURE ASSIGNMENT WITH HIGH-GAIN ERROR-ACTUATED FLIGHT CONTROL SYSTEMS

THESIS

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by
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List of Symbols

a	-	speed of sound
A	-	state matrix
A_{CL}	-	closed-loop A matrix
A_N	-	normal acceleration
$\tilde{A}$	-	perturbation state matrix
b	-	wing span / constant
B	-	input matrix
$\tilde{B}$	-	perturbation input matrix
C	-	output matrix
C_{D_O}	-	trim drag coefficient
C_{L_O}	-	trim lift coefficient
C_{M_O}	-	trim pitching moment coefficient
$\bar{c}$	-	mean aerodynamic chord
$\mathcal{C}$	-	set of complex numbers
$\tilde{C}$	-	perturbation output matrix
d	-	Riccati equation vector due to tracking
e	-	error vector
F	-	measurement matrix
g	-	acceleration due to gravity / gain parameter
$G(\ )$	-	transfer function, in terms of ( )
h	-	altitude
H	-	weighting matrix

<u>List of Symbols</u>

u_c - commanded input vector

u^* - optimal input vector

v - command vector

w - vertical perturbation velocity

$\underline{w}$ - measurement vector

x - state vector

X,Z - components of orthogonal reference frame

y - output vector

z - time integral of error vector

α - angle of attack

α_o - trim angle of attack

γ - flight path angle

Γ - asymptotic transfer function

$\tilde{\Gamma}$ - slow asymptotic transfer function

$\hat{\Gamma}$ - fast asymptotic transfer function

δ_e - elevator deflection

δ_f - flap deflection

δ_{e_c} - commanded elevator deflection

δ_{f_c} - commanded flap deflection

ε - perturbation parameter / "is a member of"

List of Symbols

I_i - identity matrix of rank i

Iy - moment of inertia about the center of mass

J - performance index

K - controller matrix / feedback gain matrix

m - elements of transducer matrix

M - Mach number / transducer matrix

P - Riccati equation solution matrix

q - pitch rate

Q - weighting matrix

$\bar{q}$ - dynamic pressure

r - desired output vector

R - weighting matrix

R^ℓ - matrix of dimension ℓ

s - LaPlace transform parameter

S - wing area / feedback gain matrix

t - independent variable, time

t_f - final time

u - forward perturbation velocity

$\underline{u}$ - input vector

U - aircraft velocity

List of Symbols

ζ - eigenvector

θ - pitch angle

Θ_o - trim flight path angle

λ - eigenvalue

ρ - air density

σ - fast pole

Σ - diagonal matrix of fast poles

τ - time dummy argument

ω - portion of null space vector

diag { } - elements of diagonal matrix

det [ ] - determinant of a matrix

N[ ] - null space of a matrix

$[\ ]^T$ - transpose of a matrix

$[\ ]^{-1}$ - inverse of a matrix

Stability Derivatives
(nondimensional)

$$C_{D_\alpha} = \frac{\partial C_D}{\partial \alpha} \qquad C_{L_{\delta_e}} = \frac{\partial C_L}{\partial \delta_e} \qquad C_{M_q} = \frac{\partial C_M}{\partial \frac{q\,\bar{c}}{2V_R}}$$

$$C_{L_\alpha} = \frac{\partial C_L}{\partial \alpha} \qquad C_{L_{\delta_f}} = \frac{\partial C_L}{\partial \delta_f} \qquad C_{M_{\delta_e}} = \frac{\partial C_M}{\partial \delta_e}$$

$$C_{L_{\dot{\alpha}}} = \frac{\partial C_L}{\partial \dot{\alpha}} \qquad C_{M_\alpha} = \frac{\partial C_M}{\partial \alpha} \qquad C_{M_{\delta_f}} = \frac{\partial C_M}{\partial \delta_f}$$

$$C_{L_q} = \frac{\partial C_L}{\partial \frac{q\bar{c}}{2V_R}} \qquad C_{M_{\dot{\alpha}}} = \frac{\partial C_M}{\partial \dot{\alpha}}$$

A dot over a symbol represents a time derivative $(\dot{\ })$.

A bar under a symbol denotes a vector quantity except as noted above.

Abstract

The theory of high-gain error-actuated feedback control, developed by Porter and Bradshaw, was applied to the design of various longitudinal decoupling flight control systems for an advanced aircraft. This aircraft incorporates both pitching moment (elevators) and direct lift (flaps) control devices, permitting the application of multivariable control theory. The controllers developed in this study utilized output feedback with proportional plus integral control to produce desirable closed-loop response with minimal interaction between outputs. Because of the structure of the system, measurement variables different from the outputs are necessary to apply this method. This report describes how entire eigenstructure assignment can be used to determine appropriate measurement equations by assigning their corresponding transmission zeros. A singular value decomposition was used to choose the eigenvectors from their permissible subspaces. The results show that these modes appear in the output response, even for very high gain. Therefore, selection of good eigenvalues/eigenvectors was shown to be crucial to the successful application of this theory. In addition, this report examined the effect of varying the other design parameters on the closed-loop system response.

USE OF ENTIRE EIGENSTRUCTURE ASSIGNMENT WITH HIGH-GAIN ERROR-ACTUATED FLIGHT CONTROL SYSTEMS

I Introduction

In the past several years, a great deal of work has been done with Control Configured Vehicles (CCV). A CCV is one in which advanced control technology as well as aerodynamics, structures, and propulsion are employed in the initial definition process (Ref 1:1). Advanced control technology, such as direct force control, can permit new and highly advantageous ways in which an aircraft maneuvers. Specifically, it is the direct force control capability of these aircraft that is addressed in this report. Direct force control, in conjunction with conventional elevator control surfaces, allows independent control of the normal force and pitching moment degrees of freedom of the aircraft. The most recent version of this type of aircraft is the AFTI-16 (Advanced Fighter Technology Integration), which is an F-16 aircraft modified to include additional control surfaces, such as canards, leading-, and trailing-edge flaps.

Background

The addition of these control surfaces allows a large number of direct force control flight modes, each of which involves different coupling between the aircraft degrees of freedom. For example, in the longitudinal axis, the modes found to be most useful are pitch pointing, vertical translation, and direct lift control. The pitch

pointing mode is effected by varying the aircraft pitch attitude while holding the flight path angle constant. The vertical translation maneuver requires varying the flight path angle while holding the pitch attitude constant. Varying flight path angle while holding the angle of attack constant causes the aircraft to exhibit the direct lift control maneuver. These flight modes are quite different than conventional aircraft modes in that they require non-interaction between specific outputs. To accomplish these unusual maneuvers, a controller must be designed which causes one output variable to track a given command while eliminating the response in other output variables. This controller, of necessity, must generate signals to deflect the two longitudinal control surfaces (flaps and elevator) in order to produce the desired output response. That is, this controller must be capable of controlling multiple-input, multiple-output (MIMO) systems.

Numerous design methods are currently in use for flight controllers. Classical control techniques are not well suited to this type of design, as they are very difficult to use with MIMO systems. Two major modern control design methods are optimal control and entire eigenstructure assignment. In the optimal control design method (Ref 12), it is desired to minimize the control effort required to track given commands. This requires minimizing the performance index

$$J = \frac{1}{2}[\underline{y}(t_f) - \underline{r}(t_f)]^T H[\underline{y}(t_f) - \underline{r}(t_f)]$$

$$+ \frac{1}{2} \int_0^{t_f} \{[\underline{y}(t) - \underline{r}(t)]^T Q(t)[\underline{y}(t) - \underline{r}(t)]$$

$$+ \underline{u}^T(t)R(t)\underline{u}(t)\}dt \qquad (1\text{-}1)$$

for the system described by

$$\dot{\underline{x}} = A\underline{x} + B\underline{u}$$
$$\underline{y} = C\underline{x} \qquad (1\text{-}2)$$

where $\underline{y}(t)$ is the output vector and $\underline{r}(t)$ is the desired value of the output vector. H, Q(t), and R(t) are symmetric, positive definite quadratic weighting matrices. The optimal control law which minimizes J is

$$\underline{u}^*(t) = K(t)\underline{x}(t) - R^{-1}(t)B^T \underline{d}(t) \qquad (1\text{-}3)$$

where the time-varying feedback gain matrix is

$$K(t) = -R^{-1}(t)B^T P(t) \qquad (1\text{-}4)$$

and P(t) is the solution of a non-linear, time-varying matrix differential equation known as a Riccati equation, defined by

$$\dot{P}(t) = -A^TP(t) - P(t)A + P(t)BR^{-1}(t)B^TP(t) - C^TQ(t)C$$
$$P(t_f) = C^THC \qquad (1\text{-}5)$$

Because the boundary condition in equation (1-5) is specified at t_f, the Riccati equation must be integrated backward in time, thus making on-line computer implementation difficult. In equation (1-3), $\underline{d}(t)$ is found from

$$-\underline{\dot{d}}(t) = A^T\underline{d}(t) - K(t)BR^{-1}(t)B^T\underline{d}(t) - C^TQ(t)C$$

$$\underline{d}(t_f) = -C^THC\underline{r}(t_f) \qquad (1\text{-}6)$$

Once again, this equation must be integrated backward in time due to the given boundary condition, thus requiring information to be known about the commands $\underline{r}(t)$ in the future. In addition, since t_f should not be much larger than the initial time (usually on the order of five seconds for aircraft maneuvers), a steady-state analysis is not theoretically in order, and the time-varying Riccati equation must be integrated. Solutions to this type of equation are extremely costly computationally and often difficult to achieve. Lastly, selection of the three weighting matrices must be made, which is the "art" of the method. All of these factors contribute to the difficulty and complexity of an optimal tracking control problem.

Entire eigenstructure assignment is based upon selection of the system closed loop eigenvalues and corresponding eigenvectors. Given a system described by the same state and output equations as before (equation (1-2)) a control law

$$\underline{u} = K\underline{x} + \underline{u}_c \qquad (1\text{-}7)$$

is assumed. Substituting this control law into the state equations yields the closed loop system

$$\underline{\dot{x}} = (A+BK)\underline{x} + B\underline{u}_c \qquad (1\text{-}8)$$

Using the gain matrix

$$K = [\underline{\omega}_1, \underline{\omega}_2, \ldots, \underline{\omega}_n][\underline{\zeta}_1, \underline{\zeta}_2, \ldots, \underline{\zeta}_n]^{-1} \tag{1-9}$$

produces a closed loop system (equation (1-8)) with eigenvalues λ_i and corresponding eigenvectors $\underline{\zeta}_i$, $i = 1,2,\ldots,n$, provided the eigenvectors $\underline{\zeta}_i$ and vectors $\underline{\omega}_i$ satisfy the relationship (Ref 6)

$$\begin{bmatrix} \underline{\zeta}_i \\ \underline{\omega}_i \end{bmatrix} \varepsilon \; N\{[A-\lambda_i I, B]\} \tag{1-10}$$

Care must be taken in selecting the eigenvectors from the null space $[A-\lambda_i I, B]$ so that the matrix $[\underline{\zeta}_1, \underline{\zeta}_2, \ldots, \underline{\zeta}_n]$ is invertible. Furthermore, these eigenvalues and eigenvectors must be chosen so that the amount of interaction between outputs is minimized through adjusting the transient responses. Some work has been done (Ref 2) on developing methods to select the eigenvectors to promote desirable response, but the procedure at this point in time is still largely trial and error.

In this report, a method designed especially to track commands and requiring relatively simple calculations (easily performed on-line) is examined. This method should greatly aid in the development of flight controllers for CCV type aircraft.

Problem Statement and Approach

It is desired to design and evaluate a flight controller (using a method proposed by Porter and Bradshaw

(Ref 3)) for an AFTI-16 aircraft which will exhibit the maneuvers of pitch pointing, vertical translation, and direct lift control. The relatively new method of controller design developed by Porter utilizes singular perturbation methods in the design of systems incorporating high-gain error-actuated controllers. The goal of this study is to minimize the amount of "tuning" required to effect the three direct force control maneuvers and to examine closely the results of varying parameters in Porter's method. To minimize the "art" involved in applying the design, the technique of entire eigenstructure assignment was used to select the closed-loop transmission zeros. The permissible subspace for selecting the eigenvectors was obtained by using a singular value decomposition (Ref 6).

The successful achievement of these maneuvers is based on several criteria. First, the closed-loop system must be stable. Secondly, all transient responses should die out after no more than three seconds (that is, achieving 98% of their steady state values). Another requirement is that overshoots of the steady state values are minimal, and the closed-loop, steady-state responses should achieve their commanded values with no observable error. Lastly, the closed-loop system should be very robust, thereby conforming to the above requirements when variations are made in the aircraft plant.

Scope and Assumptions

The purpose of this study was to thoroughly evaluate the design method proposed by Porter for CCV applications. As a result, design iterations were primarily to examine the effect of various parameters rather than to iterate to a "best" solution. To this end, only the longitudinal axis and its corresponding CCV modes were examined. The assumptions listed below were made to reduce the complexity of the model. None of these should seriously affect the conclusions reached as a result of this effort.

1. The earth is flat and non-rotating
2. The aircraft is rigid and of constant mass
3. The atmosphere is at rest with respect to the earth
4. Gravity is a constant acceleration
5. The aircraft X-Z plane is a plane of symmetry
6. The motion of the aircraft about a straight and level trim condition can be adequately modeled by a linear dynamical system of equations

Organization

First, the theory used in the design is described and the method for developing a controller discussed. Next, Chapter 3 develops the linearized models for the aircraft. Chapter 4 then presents the results of applying this method to the aircraft models. Several examples are examined and the results described in detail,

especially showing the effects of varying design parameters and the aircraft plant. Lastly, Chapter 5 discusses the results and provides recommendations for further studies.

II Theoretical Development

Porter's Method

In Porter's method (Ref 3), high-gain error-actuated analog control is used in conjunction with singular perturbation methods. First, attention is given to the rationale behind this approach. High gain frequently has the advantage of making the closed-loop system very robust, or insensitive to parameter variations within the plant. Also, high gain may be desirable from the viewpoint of disturbance rejection, since it can produce a very tight tracking loop, although this disturbance rejection aspect was not examined in this study.

Error-actuated control is the keystone of this method. The control law developed by this method automatically minimizes the difference between the commands and the outputs as the gain is increased. Therefore, tracking is accomplished when the gain is made large enough, and should become increasingly tight and non-interacting. How large the gain must be for good performance is one of the principal questions adddressed by this study.

Singular perturbation methods (Ref 4) are used to exhibit the asymptotic structure of the transfer function matrices. The method of singular perturbations can be used to examine the overall effect of letting a certain parameter become smaller and smaller. By regarding the

controller forward path gain (g) as the reciprocal of the perturbation parameter, ε, the results of singular perturbation theory can be applied, since as $\varepsilon \to 0$, $g = 1/\varepsilon \to \infty$. Using this theory, the asymptotic properties of the closed-loop transfer functions can be examined to determine the location of the closed-loop poles as the gain gets very large. These asymptotic closed-loop poles then determine the system response.

The basic plant for Porter's method may be represented by state and output equations of the respective forms

$$\begin{bmatrix} \dot{\underline{x}}_1(t) \\ \dot{\underline{x}}_2(t) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \underline{x}_1(t) \\ \underline{x}_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} \underline{u}(t) \tag{2-1}$$

and

$$\underline{y}(t) = [C_1 \quad C_2] \begin{bmatrix} \underline{x}_1(t) \\ \underline{x}_2(t) \end{bmatrix} \tag{2-2}$$

where B_2 is a square, non-singular matrix. Any system, by proper state and/or control transormations can be put into this form. For the examples of this study, such transformations were not needed since the original system of equations was already in this form. This method, as originally proposed by Porter and Bradshaw (Ref 5), requires that the matrix C_2B_2 have full rank. A later extension, which is examined in this study, compensates for a rank deficient C_2B_2 by introducing new measurement

equations. In all of the cases examined in this report, the rank of C_2B_2 is not full, so that extra plant measurements of the form

$$\underline{w}(t) = [C_1+MA_{11} \quad C_2+MA_{12}]\underline{x}(t) \tag{2-3}$$

$$= [F_1 \quad F_2] \begin{bmatrix} \underline{x}_1(t) \\ \underline{x}_2(t) \end{bmatrix} \tag{2-4}$$

were necessary. The matrix M is chosen by the designer. The high-gain error-actuated controller is governed by a control-law equation of the form

$$\underline{u}(t) = g\{K_o\underline{e}(t) + K_1\underline{z}(t)\} \tag{2-5}$$

where K_o and K_1 are controller gain matrices, $\underline{e}(t)$ is the error vector between input commands $\underline{v}(t)$ and the measurements $\underline{w}(t)$, and $\underline{z}(t)$ is the time integral of $\underline{e}(t)$.

It is desired that the input vector $\underline{u}(t)$ cause the output vector $\underline{y}(t)$ to track any constant command input vector $\underline{v}(t)$, so that

$$\lim_{t\to\infty} \{\underline{v}(t) - \underline{y}(t)\} = 0 \tag{2-6}$$

Such tracking is possible using the above controller form (equation (2-5)), which includes integral control, due to the fact that the error vector $\underline{e}(t)$ assumes the steady-state value (for arbitrary initial conditions)

$$\lim_{t\to\infty} \underline{e}(t) = \lim_{t\to\infty} \{\underline{v}(t) - \underline{w}(t)\}$$

$$= \lim_{t\to\infty} \{\underline{v}(t) - \underline{y}(t) - M(A_{11}\underline{x}_1(t) + A_{12}\underline{x}_2(t))\}$$

$$= \lim_{t\to\infty} \{\underline{v}(t) - \underline{y}(t)\} = 0$$

since $(A_{11}\underline{x}_1(t) + A_{12}\underline{x}_2(t))$ goes to zero in steady state as a result of equation (2-1). In equations (2-1), (2-2), (2-3), (2-4), and (2-5), $\underline{x}_1(t)\epsilon\ {}^{n-\ell}$, $\underline{x}_2(t)\epsilon R^{\ell}$, $\underline{u}(t)\epsilon R^{\ell}$, $\underline{y}(t)\epsilon R^{\ell}$, $\underline{w}(t)\epsilon R^{\ell}$, $A_{11}\epsilon R^{(n-\ell)x(n-\ell)}$, $A_{12}\epsilon R^{(n-\ell)x\ell}$, $A_{21}\epsilon R^{\ell x(n-\ell)}$, $A_{22}\epsilon R^{\ell x\ell}$, $B_2\epsilon R^{\ell x\ell}$, $C_1\epsilon R^{\ell x(n-\ell)}$, $C_2\epsilon R^{\ell x\ell}$, $F_1\epsilon R^{\ell x(n-\ell)}$, $F_2\epsilon R^{\ell x\ell}$, rank $C_2B_2\epsilon R$, rank $F_2B_2=\ell$, $\underline{e}(t)\epsilon R^{\ell}$, $\underline{v}(t)\epsilon R^{\ell}$, $\underline{z}(t)= \underline{z}(0) + \int_0^t \underline{e}(\tau)d\tau\epsilon R^{\ell}$, $K_o\epsilon R^{\ell x\ell}$, $K_1\epsilon R^{\ell x\ell}$, $g>0$, and $M\epsilon R^{\ell x(n-\ell)}$.

The error states are governed by the differential equations

$$\dot{\underline{z}}(t) = \underline{e}(t) = \underline{v}(t) - \underline{w}(t) = \underline{v}(t) - F_1\underline{x}_1(t) - F_2\underline{x}_2(t) \quad (2\text{-}8)$$

Substituting this expression for $\underline{e}(t)$ into equation (2-5) and then the resulting equation into (2-1), along with equation (2-8), yields the closed-loop equations

$$\begin{bmatrix} \dot{\underline{z}}(t) \\ \dot{\underline{x}}_1(t) \\ \dot{\underline{x}}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & -F_1 & -F_2 \\ 0 & A_{11} & A_{12} \\ gB_2K_1 & A_{21}-gB_2K_oF_1 & A_{22}-gB_2K_oF_2 \end{bmatrix} \begin{bmatrix} \underline{z}(t) \\ \underline{x}_1(t) \\ \underline{x}_2(t) \end{bmatrix}$$

$$+ \begin{bmatrix} I_{\ell} \\ 0 \\ gB_2K_o \end{bmatrix} \underline{v}(t) \quad (2\text{-}9)$$

and output equations

$$\underline{y}(t) = [0 \quad C_1 \quad C_2] \begin{bmatrix} \underline{z}(t) \\ \underline{x}_1(t) \\ \underline{x}_2(t) \end{bmatrix} \qquad (2\text{-}10)$$

The results of singular perturbation theory as described by Porter and Shenton (Ref 4) are used to determine the asymptotic form of the transfer function matrix, $G(\lambda)$, as $g \to \infty$. Letting the perturbation parameter $\varepsilon = 1/g$ go to zero, the following system is obtained

$$A = \begin{bmatrix} \tilde{A}_1 & \tilde{A}_2 \\ \tilde{A}_3/\varepsilon & \tilde{A}_4/\varepsilon \end{bmatrix} \quad B = \begin{bmatrix} \tilde{B}_1 \\ \tilde{B}_2/\varepsilon \end{bmatrix}$$

$$C = [\tilde{C}_1 \quad \tilde{C}_2] \qquad (2\text{-}11)$$

where the components of the asymptotic closed-loop A matrix are

$$\tilde{A}_1 = \begin{bmatrix} 0 & -F_1 \\ 0 & A_{11} \end{bmatrix} \quad \tilde{A}_2 = \begin{bmatrix} -F_2 \\ A_{12} \end{bmatrix} \qquad (2\text{-}12)$$

$$\tilde{A}_3 = [B_2K_1 \quad -B_2K_oF_1] \quad \tilde{A}_4 = [-B_2K_oF_2]$$

the B matrix components are

$$\tilde{B}_1 = \begin{bmatrix} I_\ell \\ 0 \end{bmatrix} \quad \tilde{B}_2 = [B_2K_o] \qquad (2\text{-}13)$$

and the C matrix elements are

$$\tilde{C}_1 = [0 \quad C_1] \quad \tilde{C}_2 = [C_2] \qquad (2\text{-}14)$$

As $g \to \infty$, the transfer function matrix, $G(\lambda) = C[\lambda I - A]^{-1}B$, assumes the form

$$\Gamma(\lambda) = \tilde{\Gamma}(\lambda) + \hat{\Gamma}(\lambda) \tag{2-15}$$

where

$$\tilde{\Gamma}(\lambda) = \tilde{C}_o(\lambda I_n - \tilde{A}_o)^{-1}\tilde{B}_o \tag{2-16}$$

$$\hat{\Gamma}(\lambda) = \tilde{C}_2(\lambda I_\ell - g\tilde{A}_4)^{-1} g\, \tilde{B}_2 \tag{2-17}$$

The matrices in equation (2-16) are

$$\tilde{A}_o = \tilde{A}_1 - \tilde{A}_2\tilde{A}_4^{-1}\tilde{A}_3 = \begin{bmatrix} -K_o^{-1}K_1 & 0 \\ A_{12}F_2^{-1}K_o^{-1}K_1 & A_{11} - A_{12}F_2^{-1}F_1 \end{bmatrix} \tag{2-18}$$

$$\tilde{B}_o = \tilde{B}_1 - \tilde{A}_2\tilde{A}_4^{-1}\tilde{A}_3 = \begin{bmatrix} 0 \\ A_{12}F_2^{-1} \end{bmatrix} \tag{2-19}$$

and

$$\tilde{C}_o = \tilde{C}_1 - \tilde{C}_2\tilde{A}_4^{-1}\tilde{A}_3 = [C_2F_2^{-1}K_o^{-1}K_1 \quad C_1 - C_2F_2^{-1}F_1] \tag{2-20}$$

The "slow" modes of the tracking system are defined as those which are not directly dependent on g. These are the poles of $\tilde{\Gamma}(\lambda)$ and are easily found from

$$\det [\lambda I_n - \tilde{A}_o] = 0 \tag{2-21}$$

since $\tilde{A}_o$ is block lower triangular. Simplifying this relation yields the two sets of slow poles described by

$$Z_1 = \{\lambda \in \mathcal{C} : |\lambda K_o + K_1| = 0\} \tag{2-22}$$

and

$$Z_2 = \{\lambda \varepsilon C: |\lambda I_{n-\ell} - A_{11} + A_{12}F_2^{-1}F_1| = 0\} \qquad (2\text{-}23)$$

The "fast" modes of the system (those directly dependent on g) are found from

$$\det [\lambda I_\ell - g\tilde{A}_4] = 0 \qquad (2\text{-}24)$$

so that

$$Z_3 = \{\lambda \varepsilon C: |\lambda I_\ell + gF_2B_2K_o| = 0\} \qquad (2\text{-}25)$$

In order for tracking to occur, the closed-loop system must obviously be stable, requiring that Z_1, Z_2, and Z_3 have all poles in the left-half plane. Therefore, for sufficiently large gains, the controller and transducer matrices K_o, K_1, and M must be chosen so that $Z_1 \varepsilon C^-$, $Z_2 \varepsilon C^-$, and $Z_3 \varepsilon C^-$, and the chosen M matrix must simultaneously satisfy the condition

$$\text{rank } F_2B_2 = \ell \qquad (2\text{-}26)$$

As there is no real physical insight to the best selection of M or even values of M that will satisfy the above conditions, an extension to Porter's method was developed.

Entire Eigenstructure Assignment Extension

To eliminate the necessity of the designer selecting the transducer matrix, M, while simultaneously satisfying equation (2-26) and ensuring that the poles of Z_2 are in the left-half plane, an extension to Porter's method

incorporating entire eigenstructure assignment was developed. The equation for determining the second set of slow eigenvalues (which are solely dependent on the choice of M) is

$$\det\,[\lambda I_{n-\ell}-A_{11} + A_{12}F_2^{-1}F_1] = 0 \qquad (2\text{-}27)$$

These poles correspond to closed-loop transmission zeros between the inputs $\underline{u}(t)$ and the measurements $\underline{w}(t)$. Since the measurements are being chosen by the designer, by way of choosing M, the designer is essentially free to pick those transmission zeros. This is not true for those cases where C_2B_2 has full rank, since then $F_1 = C_1$ and $F_2 = C_2$ and there is no choice in picking the transmission zeros.

Equation (2-27) can be transformed into an entire eigenstructure assignment problem by letting

$$S = F_2^{-1}F_1 \qquad (2\text{-}28)$$

Equation (2-27) then becomes a problem of assigning the eigenvalues of

$$A_{11}-A_{12}S \equiv A_{CL} \qquad (2\text{-}29)$$

Writing:

$$[\lambda I_{n-\ell}-A_{11} + A_{12}S]\ \underline{\zeta}_i = 0 \qquad (2\text{-}30)$$

where λ_i is the closed-loop eigenvalue of $A_{11}-A_{12}S$ (and incidentally of the asymptotic closed-loop system) and ζ_i is the corresponding eigenvector. Letting

$$S\underline{\zeta}_i = \underline{\omega}_i \tag{2-31}$$

equation (2-30) becomes

$$[\lambda_i I - A_{11} \quad A_{12}] \begin{bmatrix} \underline{\zeta}_i \\ \underline{\omega}_i \end{bmatrix} = 0 \tag{2-32}$$

Therefore, $[\frac{\underline{\zeta}_i}{\underline{\omega}_i}]$ lies in the null space of $[\lambda_i I - A_{11} \quad A_{12}]$. Using the method of singular value decomposition suggested by Silverthorn and Reid (Ref 6) to find the null space of $[\lambda_i I - A_{11} \quad A_{12}]$, $\underline{\zeta}_i$ and $\underline{\omega}_i$ can be calculated for any choice of closed-loop eigenvalues. Finally, the gain matrix S can be found from repeated application of equations (2-30), (2-31), and (2-32) for the n closed-loop eigenvalues. That is

$$S = [\underline{\omega}_1, \underline{\omega}_2, \ldots, \underline{\omega}_n] \; [\underline{\zeta}_1, \underline{\zeta}_2, \ldots, \underline{\zeta}_n]^{-1} \tag{2-33}$$

From equation (2-33), it is clear that the matrix $[\underline{\zeta}_1, \underline{\zeta}_2, \ldots, \underline{\zeta}_n]$ must be invertible, therefore the $\underline{\zeta}_i$'s i = 1,2,..., n must be chosen so that they are independent. Nevertheless, much design freedom still exists in choosing these eigenvectors. Some effects of choosing the $\underline{\zeta}_i$ are shown in the results section.

Having found S, the matrix M can be found from equation (2-28) and equation (2-3). By assuming that F_2^{-1} is nonsingular, from equation (2-28)

$$F_2 S = F_1 \tag{2-34}$$

(unfortunately, the method of entire eigenstructure assignment does not assure that $(F_2^{-1})^{-1}$ exists.) Using the expressions for F_1 and F_2 in equation (2-3), this becomes

$$(C_2 + MA_{12})S = (C_1 + MA_{11}) \tag{2-35}$$

This is rearranged to yield

$$M = (C_2S - C_1)(A_{11} - A_{12}S)^{-1} \tag{2-36}$$

Since the latter matrix is imply A_{CL} given by equation (2-29), it is guaranteed to be invertible as long as no eigenvalues are chosen to be zero (choosing zero for a closed-loop pole would make the system marginally stable, so this is not a problem). Now F_1 and F_2 can be calculated from equation (2-3). F_2 must be checked to insure that it is nonsingular. If it is singular, either different eigenvectors must be selected or this method is not applicable for the given system (such a case is presented in the results section). The method described above for computing M is an alternative to the method suggested by Porter (guessing M so that F_2 has full rank) since there is considerably more physical insight behind the selection of these closed-loop eigenvalues than the selection of a transducer matrix M.

In summary, to use this synthesis method, first closed-loop "slow" poles are chosen and their corresponding eigenvectors are calculated through a computationally efficient singular value decomposition (equation (2-32)).

A gain matrix S is computed (equation (2-33)), from which M, F_1, and F_2 are calculated (equations (2-36) and (2-3)).

Next, the fast modes are selected. The matrix $F_2B_2K_o$ should be chosen to be a diagonal matrix in order for increasingly non-interacting tracking to occur as $g \to \infty$. From equation (2-17), a diagonal $F_2B_2K_o$ obviously produces a diagonal transfer function $\hat{\Gamma}(\lambda)$. Therefore, letting

$$F_2B_2K_o = \Sigma = \text{diag}\,\{\sigma_1, \sigma_2, \ldots, \sigma_\ell\} \qquad (2\text{-}37)$$

the matrix K_o can be calculated for chosen $\sigma_i > 0$, $i = 1, 2, \ldots, \ell$ from

$$K_o = (F_2B_2)^{-1}\,\Sigma \qquad (2\text{-}38)$$

Finally, the first set of "slow" poles can be easily chosen by letting K_1 be a constant, positive multiple of K_o, so that the "slow" poles become the negative of the multiplicative factor. This is shown by letting $K_1 = bK_o$ in equation (2-21), so that

$$|\lambda I_\ell + K_o^{-1}(bK_o)| = |\lambda I_\ell + b| = 0 \qquad (2\text{-}39)$$

This clearly shows that $\lambda_i = -b$, $i = 1, 2, \ldots, \ell$.

Now that M, K_o, and K_1 have been calculated and all the necessary conditions for tracking satisfied, the closed-loop matrices, equations (2-9) and (2-10) can be calculated for choices of the gain parameter, g. From these equations,

the closed-loop eigenvalues for the entire system and the time histories of all states and outputs can be calculated.

III Development of the Aircraft Equations of Motion

Linearized Equations

As stated in the introduction, only the longitudinal equations of motion are analyzed here. The model of the airframe is a conventional, small-perturbation set of equations linearized about straight and level flight. The stability axis is used in this study, where the trim flight path angle (pitch angle of the stability axis), $\Theta_o = 0$. The resulting equations, in the LaPlace domain, are shown in Fig. 3-1 (Ref 7:298). These equations are transformed to the time domain and separated so that they are of the form

$$\dot{\underline{x}} = A\underline{x} + B\underline{u} \tag{3-1}$$

The results of this transformation are shown in Fig. 3-2.

Values for all the terms in the equations are presented in Appendix A. The basic, nondimensional derivatives were taken from Ref 8, and represented a flight condition for which h = 3,000 ft and M = 0.60. These values are substituted into the equations in Fig. 3-2 and are presented in Fig. 3-3. The open-loop poles for this system and their natural frequencies ω_n and damping ratio ζ are given in Table 3-1.

Short Period Approximation

For several cases examined in the next chapter, the short period approximation is used to show basic characteristics of the controller and to compare results with

$$su + g\theta = X_u u + X_w w + X_{\dot{w}} sw + X_q q + X_{\delta_e}\delta_e + X_{\delta_f}\delta_f$$

$$sw - Uq = Z_u u + Z_w w + Z_{\dot{w}} sw + Z_q q + Z_{\delta_e}\delta_e + Z_{\delta_f}\delta_f$$

$$sq = M_u u + M_w w + M_{\dot{w}} sw + M_q q + M_{\delta_e}\delta_e + M_{\delta_f}\delta_f$$

$$s\theta = q$$

Fig. 3-1. Linearized, Small-Perturbation Equations of Motion in Stability Axis.

$$\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} X_u + \frac{Z_u X_{\dot{w}}}{1-Z_{\dot{w}}} & X_w + \frac{Z_w X_{\dot{w}}}{1-Z_{\dot{w}}} & X_q + \frac{X_{\dot{w}}(U+Z_q)}{1-Z_{\dot{w}}} & -g \\ \frac{Z_u}{1-Z_{\dot{w}}} & \frac{Z_w}{1-Z_{\dot{w}}} & \frac{U+Z_q}{1-Z_{\dot{w}}} & 0 \\ M_u + \frac{Z_u M_{\dot{w}}}{1-Z_{\dot{w}}} & M_w + \frac{Z_w M_{\dot{w}}}{1-Z_{\dot{w}}} & M_q + \frac{M_{\dot{w}}(U+Z_q)}{1-Z_{\dot{w}}} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix}$$

$$+ \begin{bmatrix} X_{\delta_e} + \frac{X_{\dot{w}} Z_{\delta_e}}{1-Z_{\dot{w}}} & X_{\delta_f} + \frac{X_{\dot{w}} Z_{\delta_f}}{1-Z_{\dot{w}}} \\ \frac{Z_{\delta_e}}{1-Z_{\dot{w}}} & \frac{Z_{\delta_f}}{1-Z_{\dot{w}}} \\ M_{\delta_e} + \frac{M_{\dot{w}} Z_{\delta_e}}{1-Z_{\dot{w}}} & M_{\delta_f} + \frac{M_{\dot{w}} Z_{\delta_f}}{1-Z_{\dot{w}}} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_e \\ \delta_f \end{bmatrix}$$

Fig. 3-2. Differential Equations of Motion (Stability Axis) in Matrix Form.

$$\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -0.01505 & -0.09135 & 0 & -32.174 \\ -0.09694 & -1.3411 & 658.27 & 0 \\ 0.00018 & 0.06524 & -0.86939 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix}$$

$$+ \begin{bmatrix} -2.5157 & -13.136 \\ -111.97 & -166.68 \\ -17.251 & -1.5766 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_e \\ \delta_f \end{bmatrix}$$

Fig. 3-3. Linear Model for Basic Aircraft

$$\begin{bmatrix} \dot{\theta} \\ \dot{q} \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -0.86939 & 43.233 \\ 0 & 0.99335 & -1.3411 \end{bmatrix} \begin{bmatrix} \theta \\ q \\ \alpha \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 \\ -17.251 & -1.5766 \\ -0.16897 & -0.25183 \end{bmatrix} \begin{bmatrix} \delta_e \\ \delta_f \end{bmatrix}$$

Fig. 3-4. Linear Model for Short Period Approximation with α Replacing w

Table 3-1. Eigenvalues, Natural Frequencies, and Damping Ratios for System Shown in Fig. 3-3.

Eigenvalue	Natural Frequency (rad/sec)	Damping Ratio
$-0.00757 \pm 0.068j$	0.0684	0.1106
5.453	--	--
-7.663	--	--

those from an example in a paper by Porter and Bradshaw (Ref 9). In the short period approximation, the forward perturbation speed, u, is assumed to be zero. This eliminates the phugoid mode and replaces it with one pole at the origin. Also, the variable w (the vertical perturbation speed) is transformed to α (angle of attack) by the relationship

$$\alpha = w/U \tag{3-2}$$

where U is the aircraft velocity. These equations are shown in Fig. 3-4 and the open-loop poles, natural frequencies, and damping ratios as shown in Table 3-2.

Table 3-2. Eigenvalues for the System Shown in Fig. 3-4

Eigenvalue
0
5.453
-7.663

Control Actuator Dynamics

In several test cases, the effects of control actuator dynamics were evaluated. By modeling these actuators, choosing values similar to those used in fighter aircraft, the controller can not generate instantaneous deflections of the control surfaces. This factor could be extremely important in the evaluation of high-gain controllers. The simple model used for the actuator dynamics was

$$\frac{\delta}{\delta_c} = \frac{20}{s+20} \tag{3-3}$$

In the time domain, for the elevator and flap control surfaces, the linear model is

$$\begin{bmatrix} \dot{\delta}_e \\ \dot{\delta}_f \end{bmatrix} = \begin{bmatrix} -20 & 0 \\ 0 & -20 \end{bmatrix} \begin{bmatrix} \delta_e \\ \delta_f \end{bmatrix} + \begin{bmatrix} 20 & 0 \\ 0 & 20 \end{bmatrix} \begin{bmatrix} \delta_{e_c} \\ \delta_{f_c} \end{bmatrix} \tag{3-4}$$

Letting δ_e and δ_f be additional aircraft states, and δ_{e_c} and δ_{f_c} be the system inputs, the plant state equations for the full model and the short period approximation are shown in Figs. 3-5 and 3-6, respectively. The eigenvalues for these plants are the same as for their unaugmented counterparts, except two poles at -20 are added in each case.

$$\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \\ \dot{\delta}_e \\ \dot{\delta}_f \end{bmatrix} = \begin{bmatrix} -0.01505 & -0.01935 & 0 & -32.174 & -2.5157 & -13.136 \\ -0.09694 & -1.3411 & 658.27 & 0 & -111.97 & -166.68 \\ 0.00018 & 0.06524 & -0.86939 & 0 & -17.251 & -1.5766 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -20 & 0 \\ 0 & 0 & 0 & 0 & 0 & -20 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta \\ \delta_e \\ \delta_f \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 20 & 0 \\ 0 & 20 \end{bmatrix} \begin{bmatrix} \delta_{e_C} \\ \delta_{f_C} \end{bmatrix}$$

Fig. 3-5. Full Model Aircraft State Equations with Control Actuators

$$\begin{bmatrix} \dot{\theta} \\ \dot{q} \\ \dot{\alpha} \\ \dot{\delta}_e \\ \dot{\delta}_f \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & -0.86939 & 43.233 & -17.251 & -1.5766 \\ 0 & 0.99335 & -1.3411 & -0.16897 & -0.25183 \\ 0 & 0 & 0 & -20 & 0 \\ 0 & 0 & 0 & 0 & -20 \end{bmatrix} \begin{bmatrix} \theta \\ q \\ \alpha \\ \delta_e \\ \delta_f \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 20 & 0 \\ 0 & 20 \end{bmatrix} \begin{bmatrix} \delta_{e_C} \\ \delta_{f_C} \end{bmatrix}$$

Fig. 3-6. Short Period Approximation Aircraft State Equations with Control Actuators

IV. Discussion of Results

System A - Short Period Approximation With No Actuators

The first group of test cases presented in this study deals with designing controllers for the short period approximation model without control actuator dynamics shown in Fig. 3-4. It is easily seen that, for these cases,

$$A_{11} = [0] \qquad A_{12} = [1 \quad 0]$$

$$A_{12} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad A_{22} = \begin{bmatrix} -0.86939 & 43.233 \\ 0.99335 & -1.3411 \end{bmatrix} \tag{4-1}$$

$$B_1 = [0 \quad 0] \quad B_2 = \begin{bmatrix} -17.251 & -1.5766 \\ -0.16897 & -0.25183 \end{bmatrix} \tag{4-2}$$

In order to effect the pitch pointing and vertical translation modes, the output matrix for the output $\underline{y} = [\theta \ \gamma]^T$ is

$$[C_1 \vdots C_2] = \left[\begin{array}{c|cc} 1 & 0 & 0 \\ 1 & 0 & -1 \end{array}\right] \tag{4-3}$$

To effect the direct lift control mode, the output matrix

$$[C_1 \vdots C_2] = \left[\begin{array}{c|cc} 1 & 0 & 0 \\ 0 & 0 & -1 \end{array}\right] \tag{4-4}$$

is required for $y = [\gamma \ \alpha]^T$. From equations (4-3) and (4-4) it is obvious that C_2 is rank deficient in both cases, so that extra plant measurements are necessary.

Since the system is augmented with two integrators in order to achieve the desired tracking properties, the composite system is of order five. Consequently, there

are five closed-loop eigenvalues. In all of these test cases only one slow eigenvalue must be chosen, since A_{11} is simply a scalar. Because there are two system inputs, and since the associated closed-loop eigenvector is a linear combination of the vectors lying in the null space of $[\lambda_1 I - A_{11}, A_{12}]$, two constant multipliers (called eigenvector multipliers) must be chosen to form the eigenvector. Each test case in this group is evaluated separately.

Case A-1

In this case (-4+j0) is selected for the slow eigenvalue for the sake of comparison with Porter's example in Ref 9. The system output equations are those of equation (4-3). Fig. 4-1 shows the M, F_1, F_2, F_2B_2, Σ, K_o, and K_1 matrices (henceforth called the system matrices) for the choice of {1,0} for the eigenvector multipliers. Also shown in Table 4-1 are the closed-loop eigenvalue locations as the gain parameter, g, is increased (referred to as eigenvalue movements).

Pitch pointing is effected with the command vector

$$\underline{v} = [1 \quad 0]^T \text{ deg} \tag{4-5}$$

As g is increased, the system goes from instability to stability (as one of the open-loop eigenvalues is in the right-half plane), and once stable, increasing g reduces the settling times and final value overshoots. Further comment on the effects of g is presented in the conclusions section. The time responses for g = 100 are shown in Fig. 4-2. From this figure, it is seen that $y_1 = \theta$ achieves

$$M = \begin{bmatrix} 0.25 \\ 0.25 \end{bmatrix} \qquad [F_1 \mid F_2] = \left[\begin{array}{c|cc} 1 & 0.25 & 0 \\ 1 & 0.25 & -1 \end{array}\right]$$

$$F_2 B_2 = \begin{bmatrix} -4.3128 & -0.3942 \\ -4.1438 & -0.1423 \end{bmatrix}$$

$$\Sigma = \text{diag}\ \{1,1\}$$

$$K_o = \begin{bmatrix} 0.1396 & -0.3866 \\ -4.0646 & 4.2303 \end{bmatrix} = K_1$$

Fig. 4-1. System Matrices for Case A-1, Multipliers {1,0}

Table 4-1. Eigenvalue Movements for Case A-1, Multipliers {1,0}

gain, g	Eigenvalues				
0	(0,0)	(0,0)	(0,0)	(5.45,0)	(-7.66,0)
1	(-0.26,0.21)	(-0.26,-0.21)	(1.23,0)	(3.42,0)	(-8.34,0)
1.225	(-0.30,0.24)	(-0.30,-0.24)	(2.22,0.14)	(2.22,-0.14)	(-8.50,0)
5	(-0.65,0.32)	(-0.65,-0.32)	(-.30,4.05)	(0.30,-4.05)	(-11.51,0)
10	(-0.83,0.26)	(-0.83,-0.26)	(-2.27,5.27)	(-2.27,-5.27)	(-16.01,0)
23.18	(-0.95,0.15)	(-0.95,-0.15)	(-8.98,0.09)	(-8.98,-0.09)	(-28.71,0)
25	(-0.96,0.14)	(-0.96,-0.14)	(-6.66,0)	(-13.13,0)	(-30.5,0)
50	(-0.99,0.07)	(-0.99,-0.07)	(-4.57,0)	(-40.38,0)	(-55.28,0)
100	(-1.00,0.04)	(-1.00,-0.04)	(-4.22,0)	(-90.84,0)	(-105.16,0)
1000	(-1.00,0.00)	(-1.00,-0.00)	(-4.02,0)	(-991.2,0)	(-1005.0,0)
∞	(-1,0)	(-1,0)	(-4,0)	(-∞,0)	(-∞,0)

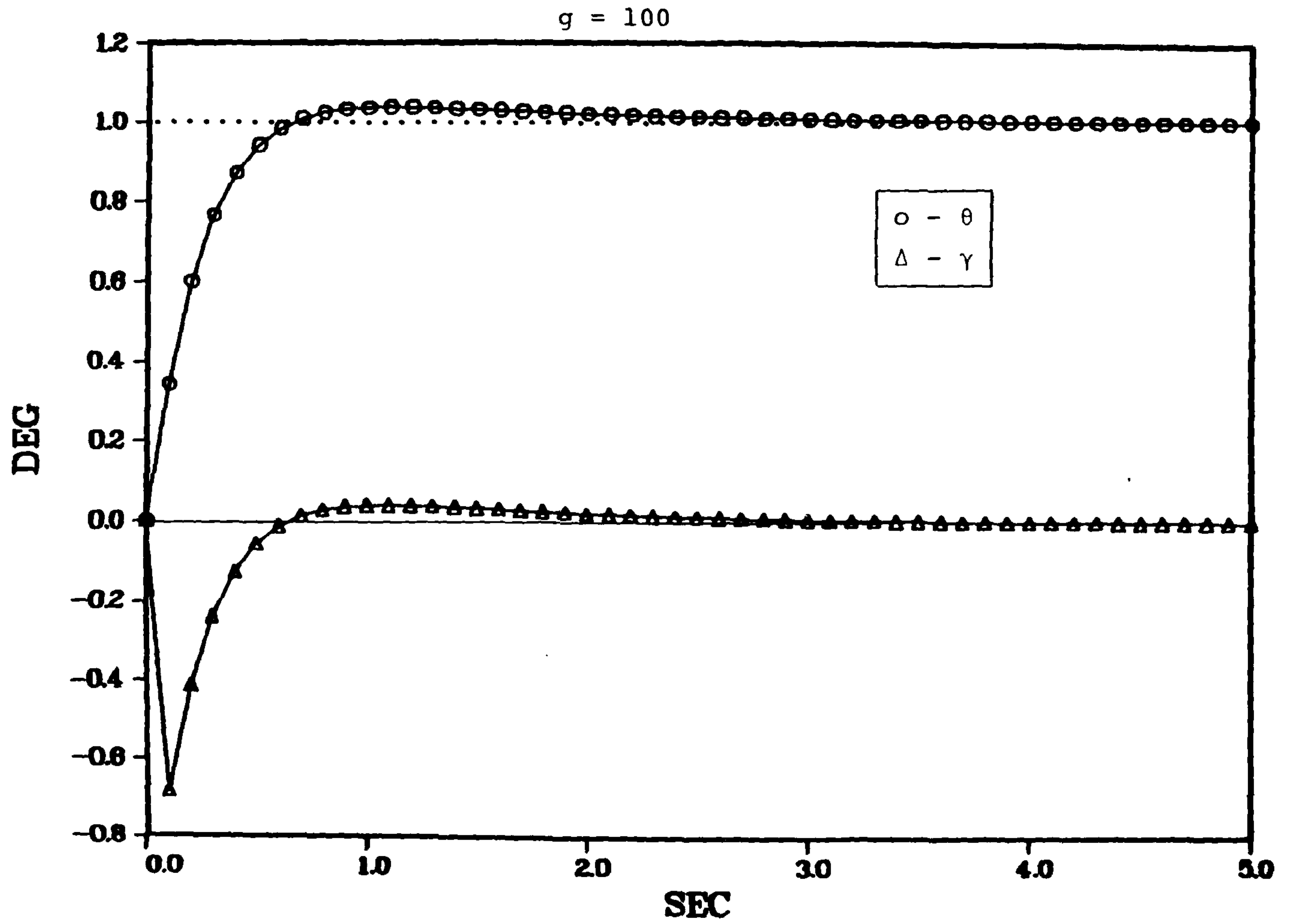

Fig. 4-2. Time Responses for Case A-1, Pitch Pointing Maneuver with Multipliers {1,0}

its commanded value quickly with little overshoot, but $y_2 = \gamma$ has a large perturbation and is not held to the desired zero value. Commanding

$$\underline{v} = [0 \quad 1]^T \text{ deg} \tag{4-6}$$

effects the vertical translation maneuver, and the results for g = 100 are shown in Fig. 4-3. Here, y_1, is held to its commanded value of zero quite well, but y_2 rises extremely rapidly and then undershoots slightly before achieving steady state.

In an attempt to improve the response, the eigenvector was changed. As this system is quite simple it is possible to calculate the required eigenvector multipliers to achieve the M matrix used by Porter (Ref 9). Using eigenvector multipliers {1,-0.25} yields the system matrices and eigenvalue movements shown in Fig. 4-4 and Table 4-2, respectively. The resulting time histories for $\underline{v} = [1 \quad 0]^T$deg and $\underline{v} = [0 \quad 1]^T$deg are shown in Figs. 4-5 and 4-6, respectively, for g = 100. The pitch pointing maneuver is now effected quite well, but the vertical translation mode is still extremely fast. This problem can be rectified by using a "ramped-up" input as Porter did, in which case these results match Porter's quite well.

Case A-2

Once again (-4+j0) is chosen for the slow eigenvalue and in this case the output equations of equation (4-4) are used so that the direct lift control maneuver is

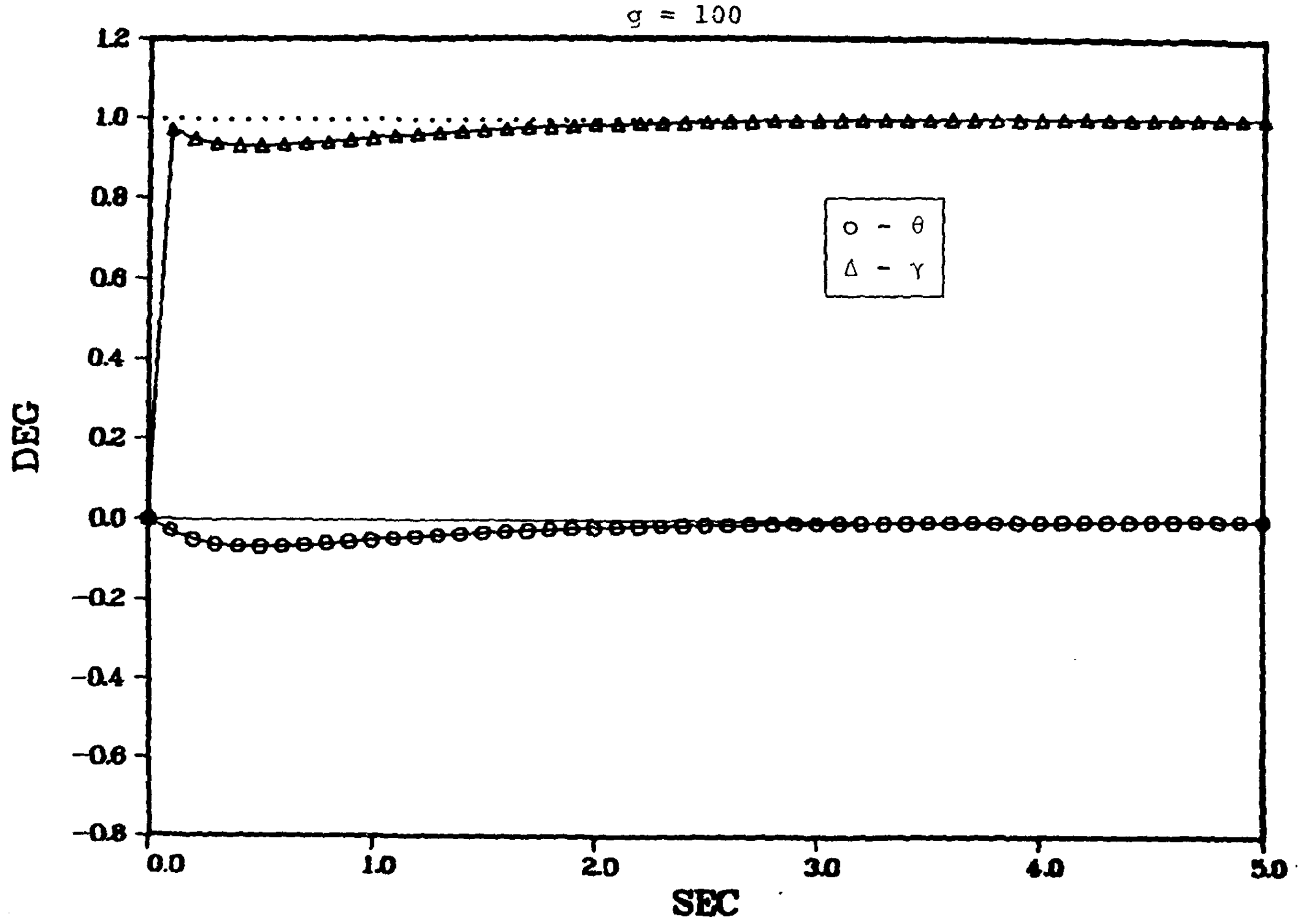

Fig. 4-3. Time Responses for Case A-1, Vertical Translation Maneuver with Multipliers {1,0}

$$M = \begin{bmatrix} 0.25 \\ -0.0077 \end{bmatrix} \qquad [F_1 \,\vdots\, F_2] = \left[\begin{array}{c:cc} 1 & 0.25 & 0 \\ 1 & -0.0077 & -1 \end{array}\right]$$

$$F_2B_2 = \begin{bmatrix} -4.3128 & -0.3942 \\ -4.1438 & -0.1423 \end{bmatrix}$$

$$\Sigma = \text{diag } \{1,1\}$$

$$K_o = \begin{bmatrix} -0.2589 & -0.3866 \\ 0.2959 & 4.2303 \end{bmatrix} = K_1$$

Fig. 4-4. System Matrices for Case A-1, Multipliers {1,-0.25}

Table 4-2. Eigenvalue Movements for Case A-1, Multipliers {1,-0.25}

gain, g	Eigenvalues				
0	(0,0)	(0,0)	(0,0)	(5.45,0)	(-7.66,0)
1	(-0.47,0.85)	(-0.47,-0.85)	(0.12,0)	(4.55,0)	(-7.94,0)
3.745	(-1.74,0.66)	(-1.74,-0.66)	(1.34,0.08)	(1.34,-0.08)	(-8.91,0)
4.3	(-1.91,0)	(-2.08,0)	(1.16,0.83)	(1.16,-0.83)	(-9.14,0)
5	(-1.45,0)	(-3.18,0)	(0.94,1.19)	(0.94,-1.19)	(-9.45,0)
10	(-1.14,0)	(-8.01,0)	(-0.33,1.85)	(-0.33,-1.85)	(-12.40,0)
25	(-1.05,0)	(-22.08,0)	(-1.73,1.11)	(-1.73,-1.11)	(-22.08,0)
30	(-1.04,0)	(-26.94,0)	(-1.88,0.83)	(-1.88,-0.83)	(-30.47,0)
50	(-1.02,0)	(-46.65,0)	(-1.46,0)	(-2.86,0)	(-50.22,0)
100	(-1.01,0)	(-96.45,0)	(-1.17,0)	(-3.52,0)	(-100.07,0)
∞	(-1,0)	$(-\infty,0)$	(-1,0)	(-4,0)	$(-\infty,0)$

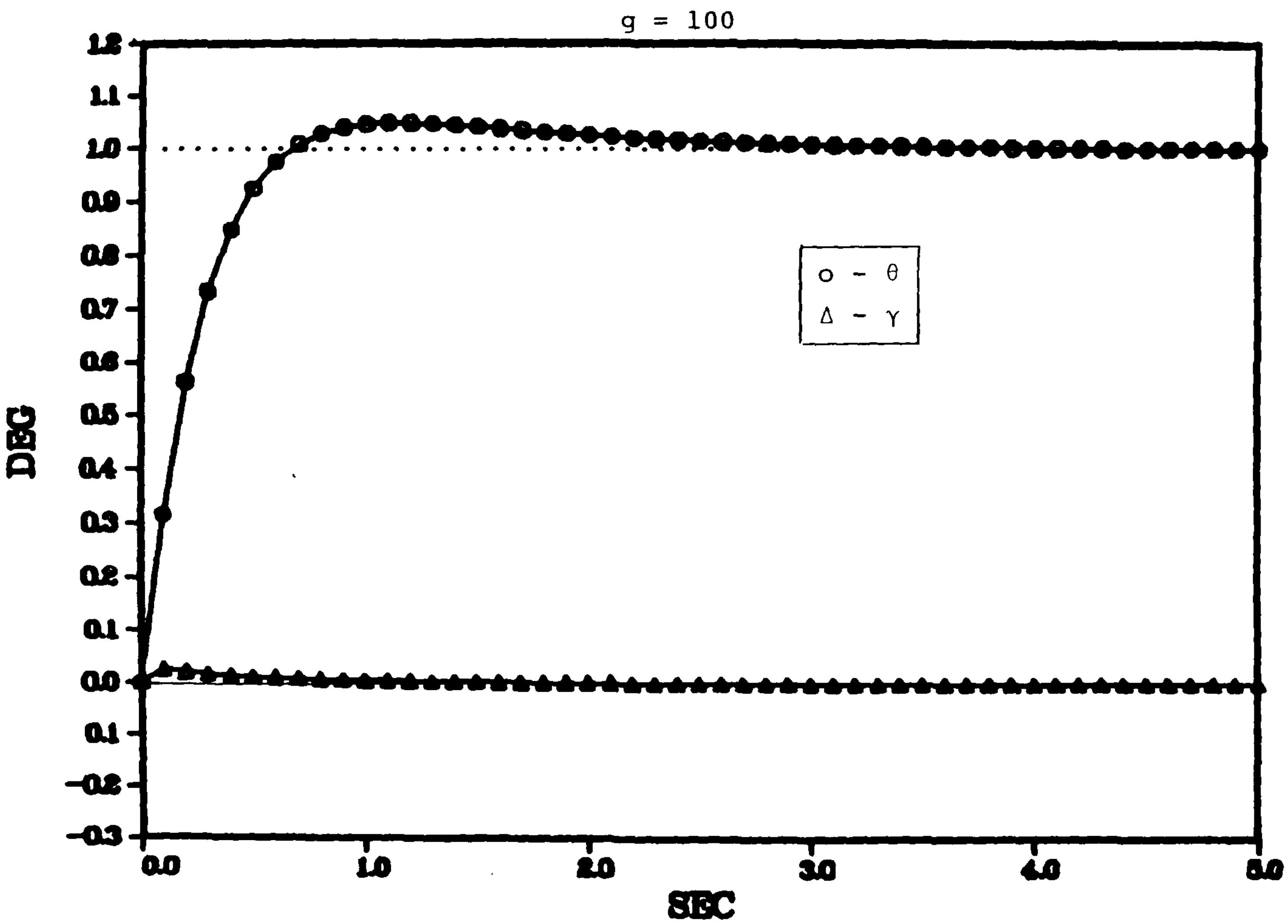

Fig. 4-5. Time Responses for Case A-1, Pitch Pointing Maneuver with Multipliers {1,-0.25}

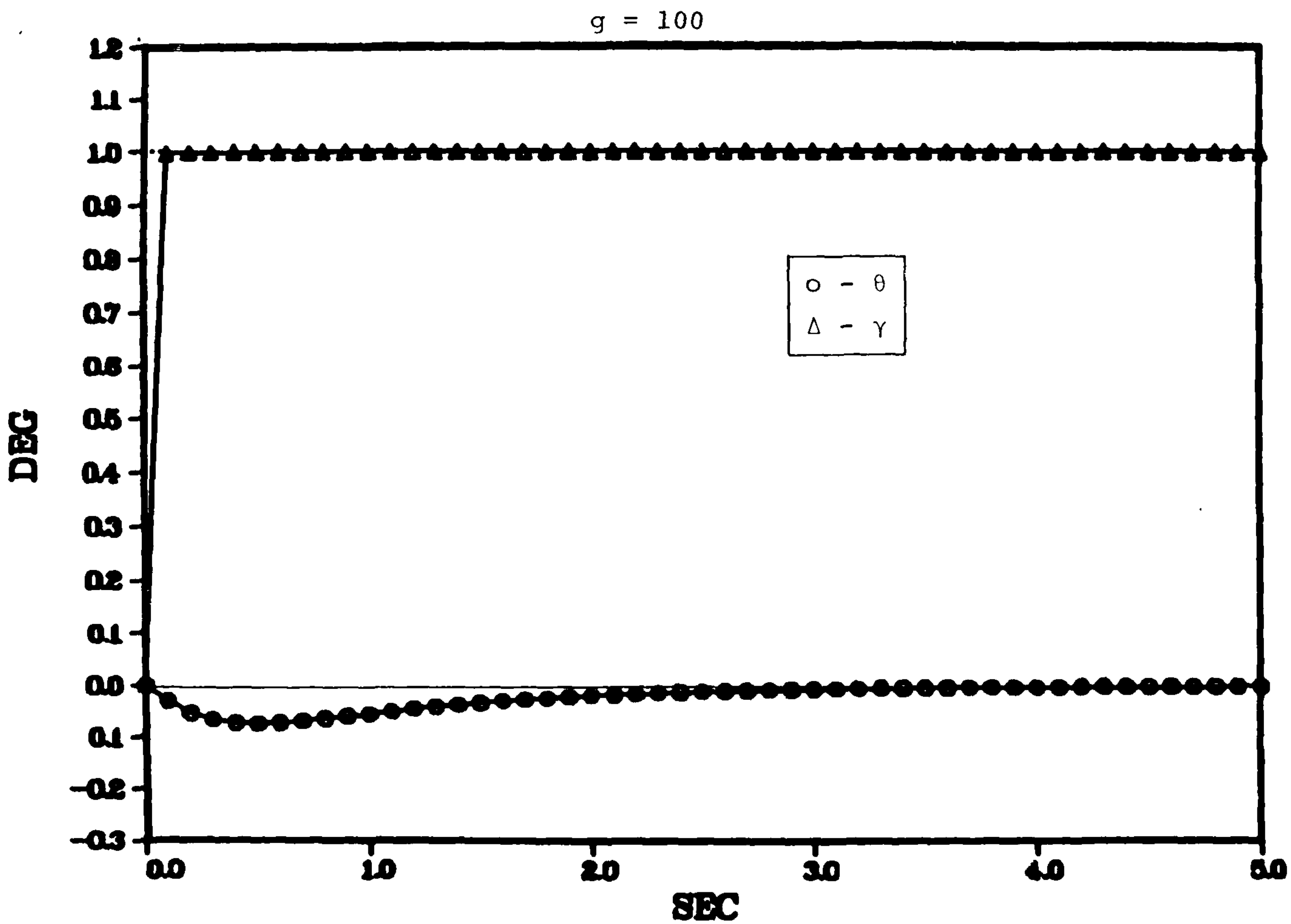

Fig. 4-6. Time Responses for Case A-1, Vertical Translation Maneuver with Multipliers {1,-0.25}

simulated by using the command vector $\underline{v} = [1\ 0]^T$deg. Choosing the multipliers {1,0} yields the system matrices shown in Fig. 4-7. The eigenvalue movements are identical to those of Table 4-1. The time responses are shown in Fig. 4-8 for g = 100, and clearly show the maneuver is satisfactorily effected.

<u>Case A-3</u>

In this case (-4+j0) is the chosen eigenvalue again, but here the controller matrix K_1 is chosen to be twice the K_o matrix, yielding {(-2+j0); (-2+j0)} for the asymptotic values for the first set of slow eigenvalues rather than {(-1+j0); (-1+j0)} as in the two previous cases. The output equations of equation (4-3) are used with the command vectors of equations (4-5) and (4-6). Four different choices of eigenvectors are made to examine the results of changing eigenvectors. The choices of the multipliers and a summary of their corresponding results are shown in Table 4-3. The results indicate that the larger the values of the elements in the matrix F_2B_2, the worse the response. The last case is shown in detail in Figs. 4-9 and Table 4-4 and its corresponding time responses shown in Figs. 4-10 and 4-11 for g = 100. These responses are very similar to those of Case A-1 for the same eigenvector except that in this case the pitch angle θ overshoots slightly less and the settling times are slightly smaller.

$$M = \begin{bmatrix} 0.25 \\ 0 \end{bmatrix} \qquad [F_1 \mid F_2] = \left[\begin{array}{c|cc} 1 & 0.25 & -1 \\ 0 & 0 & 1 \end{array}\right]$$

$$F_2 B_2 = \begin{bmatrix} -4.1438 & -0.1423 \\ -0.1690 & -0.2518 \end{bmatrix}$$

$$\Sigma = \text{diag } \{1,1\}$$

$$K_o = \begin{bmatrix} -0.2470 & 0.1396 \\ 0.1657 & -4.0646 \end{bmatrix} = K_1$$

Fig. 4-7. System Matrices for Case A-2, Multipliers {1,0}

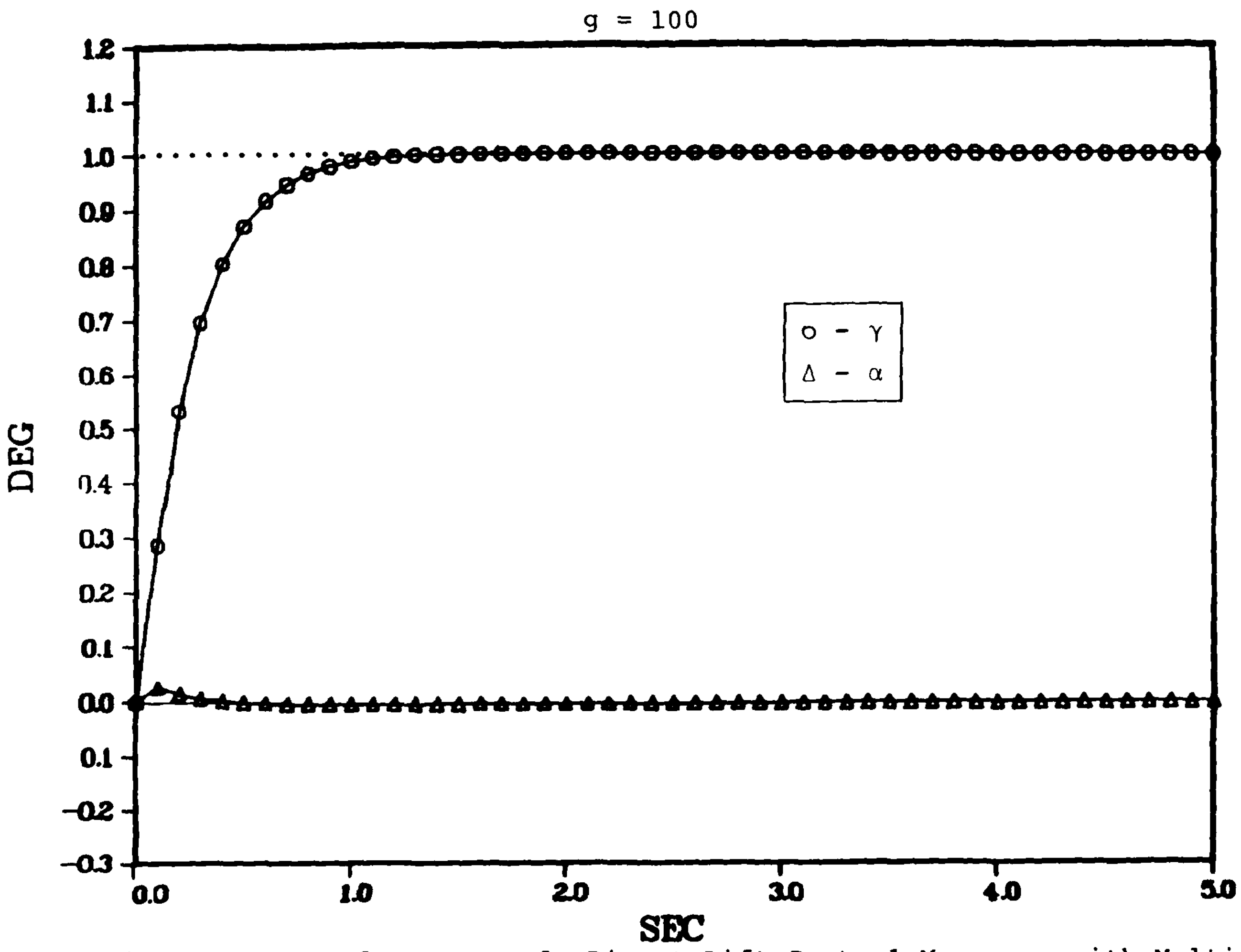

Fig. 4-8. Time Responses for Case A-2, Direct Lift Control Maneuver with Multipliers {1,0}

Table 4-3. System Matrices and Maximum Interactions for Several Choices of Eigenvectors, Case A-3.

Multipliers	M Matrix	F_2B_2 Matrix	Max Value of γ(deg)
{1,1}	$\begin{bmatrix} 0.25 \\ 1.2808 \end{bmatrix}$	$\begin{bmatrix} -4.3128 & -0.3942 \\ -21.926 & -1.7674 \end{bmatrix}$	-3.097
{10,1}	$\begin{bmatrix} 0.25 \\ 0.3531 \end{bmatrix}$	$\begin{bmatrix} -4.3128 & -0.3942 \\ -5.922 & -0.3046 \end{bmatrix}$	-0.9631
{1,0}	$\begin{bmatrix} 0.25 \\ 0.25 \end{bmatrix}$	$\begin{bmatrix} -4.3128 & -0.3942 \\ -4.1438 & -0.1423 \end{bmatrix}$	-0.6901
{1,-0.25}	$\begin{bmatrix} 0.25 \\ -0.0077 \end{bmatrix}$	$\begin{bmatrix} -4.3128 & -0.3942 \\ 0.3017 & 0.264 \end{bmatrix}$	0.0258

$$M = \begin{bmatrix} 0.25 \\ -0.0077 \end{bmatrix} \qquad [F_1 \,|\, F_2] = \left[\begin{array}{c|cc} 1 & 0.25 & 0 \\ 1 & -0.0077 & -1 \end{array}\right]$$

$$F_2 B_2 = \begin{bmatrix} -4.3128 & -0.3942 \\ 0.3017 & -0.264 \end{bmatrix}$$

$$\Sigma = \mathrm{diag}\,\{1,1\}$$

$$K_o = \begin{bmatrix} -0.2589 & -0.3866 \\ -0.2959 & 4.2303 \end{bmatrix} = \frac{1}{2} K_1$$

Fig. 4-9. System Matrices for Case A-3, Multipliers {1,-0.25}

Table 4-4. Eigenvalue Movements for Case A-3, Multipliers {1,-0.25}

gain,g	Eigenvalues				
0	(0,0)	(0,0)	(0,0)	(4.45,0)	(-7.66,0)
1	(-0.47,1.28)	(-0.47,-1.28)	(0.25,0)	(4.39,0)	(-7.90,0)
2.68	(-1.26,1.85)	(-1.26,-1.85)	(1.65,0.13)	(1.65,-0.13)	(-8.37,0)
5	(-2.32,1.96)	(-2.32,-1.96)	(0.81,2.02)	(0.81,-2.02)	(-9.18,0)
8.64	(-3.97,0.08)	(-3.97,-0.08)	(-0.30,2.61)	(-0.30,-2.61)	(-10.94,0)
10	(-2.91,0)	(-6.25,0)	(-0.64,2.66)	(-0.64,2.66)	(-11.77,0)
25	(-2.21,0)	(-20.87,0)	(-2.26,1.92)	(-2.26,-1.92)	(-24.61,0)
50	(-2.09,0)	(-45.54,0)	(-2.70,1.12)	(-2.70,-1.12)	(-49.19,0)
100	(-2.05,0)	(-95.39,0)	(-2.86,0.29)	(-2.86,-0.29)	(-99.05,0)
500	(-2.00,0)	(-995.29,0)	(-2.04,0)	(-3.93,0)	(-998.95,0)
∞	(-2,0)	(- ∞,0)	(-2,0)	(-4,0)	(- ∞,0)

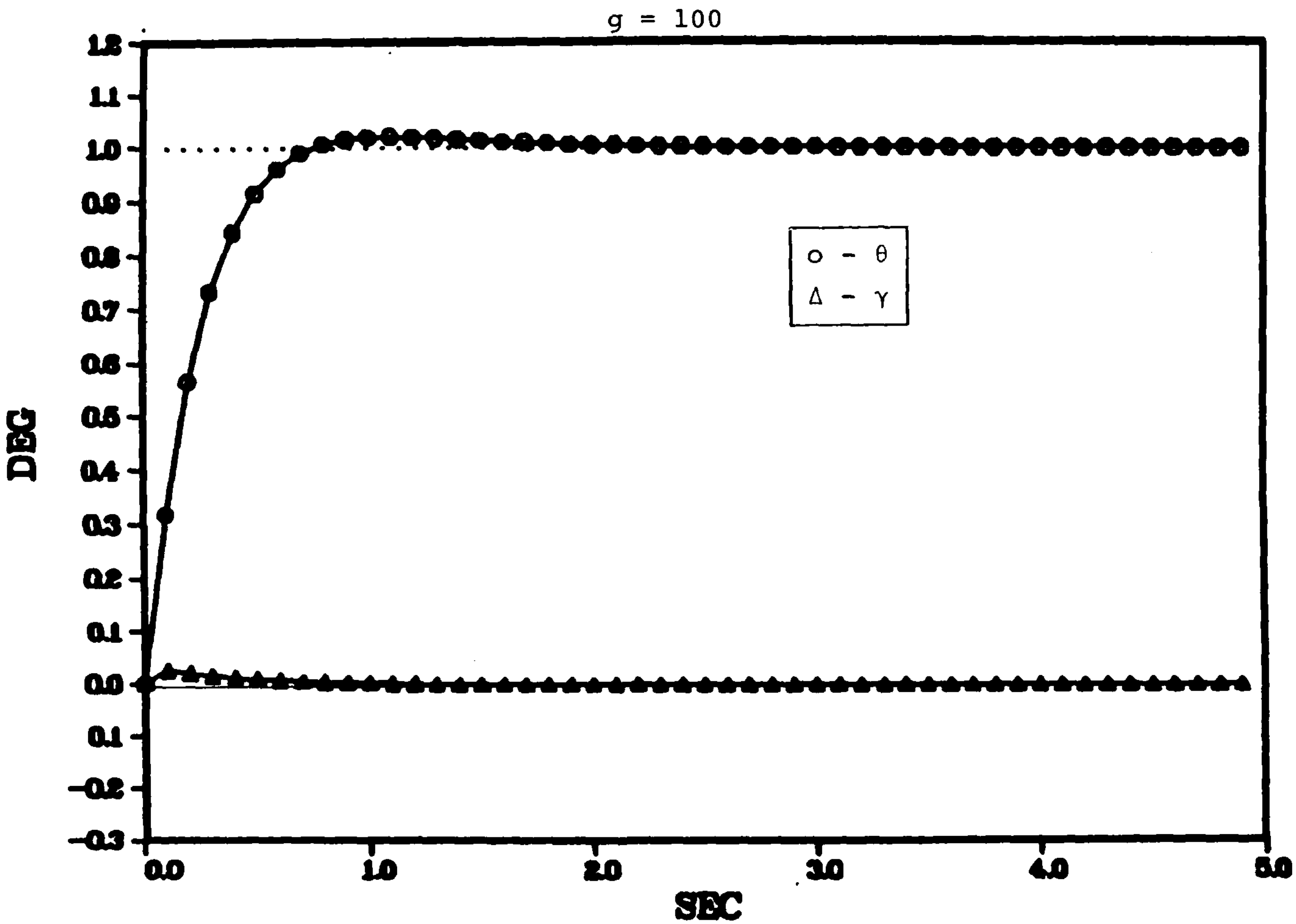

Fig. 4-10. Time Responses for Case A-3, Pitch Pointing Maneuver with Multipliers {1,-0.25}

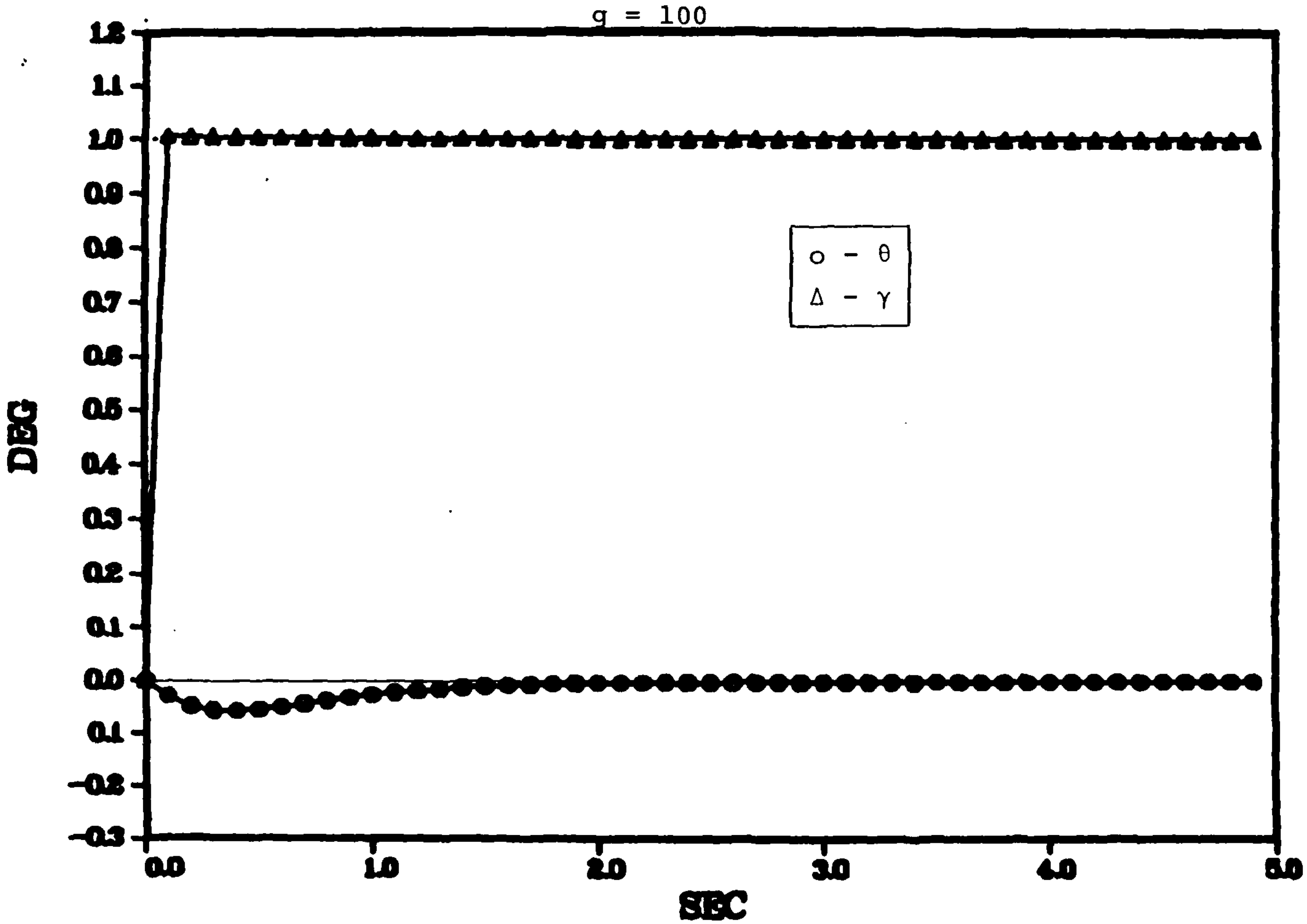

Fig. 4-11. Time Responses for Case A-3, Vertical Translation Maneuver with Multipliers {1,-0.25}

Case A-4

This case was used solely for comparison with Case A-2, and therefore is identical except for the ratio between K_o and K_1. The results are given in Fig. 4-12, Table 4-5, and Fig. 4-13. There is no notable difference in the time responses of this case versus those of Case A-2.

Case A-5

In this test case, the output equations given by:

$$[C_1 \mid C_2]\underline{x}(t) = [\theta \;\; q]^T = \left[\begin{array}{c|cc} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right]\underline{x}(t) \tag{4-7}$$

are used to show an interesting aspect of this method. Using the method developed in this report, for all choices of slow eigenvalues and corresponding eigenvectors the rank of F_2 is never full. Expanding the theory developed by Porter for this example shows the reason for this. Using the matrices given in equations (4-1), (4-2), and (4-7) with the M matrix represented by

$$M = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} \tag{4-8}$$

the following F_1 and F_2 matrices are found

$$F_1 = C_1 + MA_{11} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} [0] = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{4-9}$$

$$M = \begin{bmatrix} 0.25 \\ 0 \end{bmatrix} \qquad [F_1 \mid F_2] = \left[\begin{array}{c|cc} 1 & 0.25 & -1 \\ 1 & 0 & 1 \end{array}\right]$$

$$F_2B_2 = \begin{bmatrix} -4.3128 & -0.1423 \\ -0.1600 & -0.2518 \end{bmatrix}$$

$$\Sigma = \text{diag } \{1,1\}$$

$$K_o = \begin{bmatrix} -0.247 & 0.1396 \\ 0.1657 & -4.0646 \end{bmatrix} = \frac{1}{2} K_1$$

Fig. 4-12. System Matrices for Case A-4, Multipliers {1,0}

Table 4-5. Eigenvalue Movements for Case A-4, Multipliers {1,0}

gain, g	Eigenvalues				
0	(0,0)	(0,0)	(0,0)	(5.45,0)	(-7.66,0)
0.94	(-0.38,0.38)	(-0.38,-0.38)	(2.44,0.06)	(2.44,-0.06)	(-8.21,0)
2.5	(-0.72,0.58)	(-0.72,-0.58)	(1.73,3.12)	(1.73,-3.12)	(-9.23,0)
5	(-1.08,0.67)	(-1.08,-0.67)	(0.52,4.71)	(0.52,-4.71)	(-11.09,0)
10	(-1.46,0.62)	(-1.46,-0.62)	(-1.96,6.11)	(-1.96,-6.11)	(-15.37,0)
25	(-1.82,0.40)	(-1.82,-0.40)	(-9.46,2.76)	(-9.46,-2.76)	(-29.64,0)
26.3	(-1.83,0.39)	(-1.83,-0.39)	(-9.63,0)	(-10.60,0)	(-30.92,0)
50	(-1.94,0.23)	(-1.94,-0.23)	(-4.95,0)	(-39.04,0)	(-54.35,0)
100	(-1.98,0.13)	(-1.98,-0.13)	(-4.35,0)	(-89.71,0)	(-104.2,0)
∞	(-2,0)	(-2,0)	(-4,0)	(-∞,0)	(-∞,0)

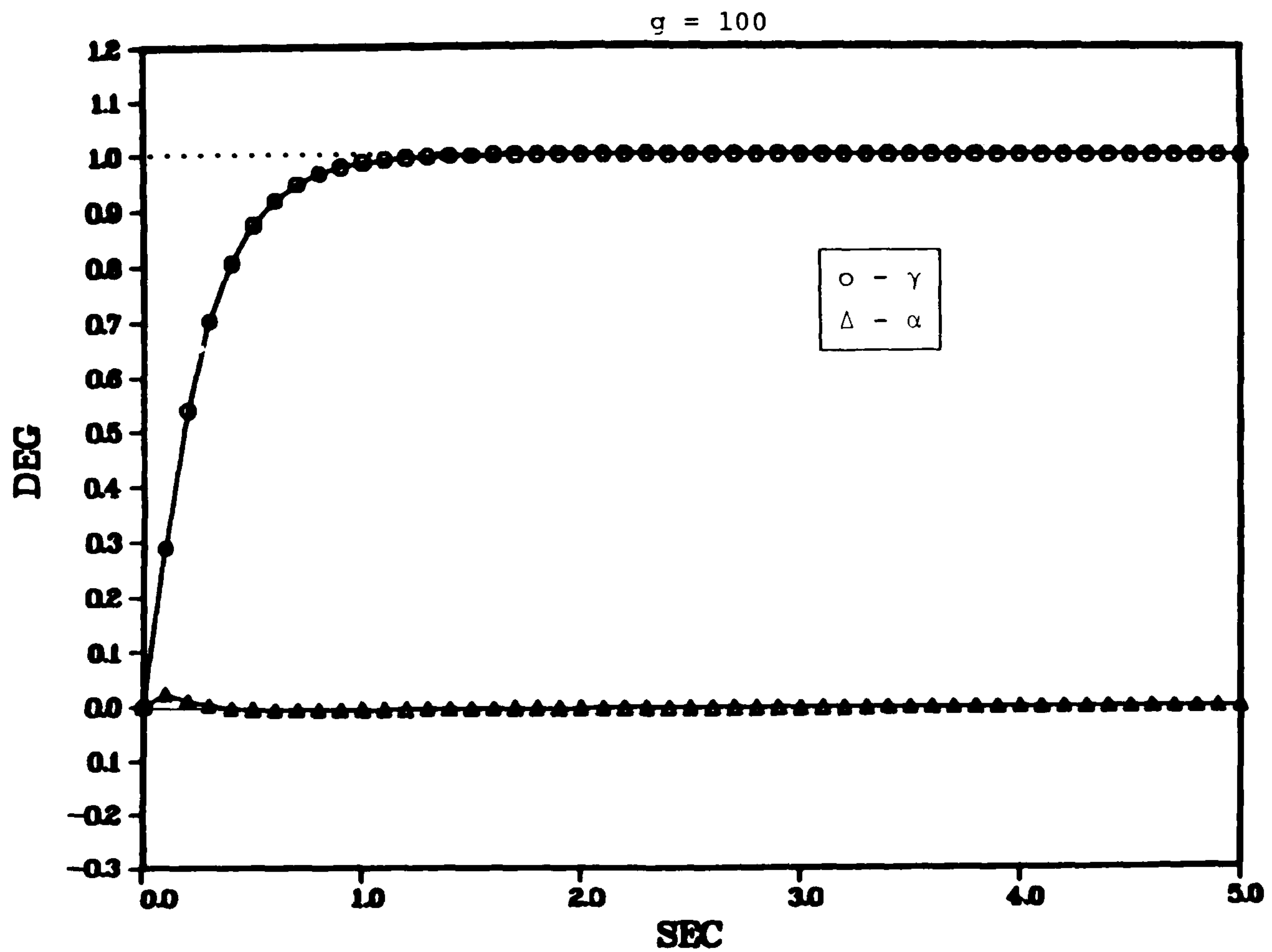

Fig. 4-13. Time Responses for Case A-4, Direct Lift Control Maneuver with Multipliers {1,0}

$$F_2 = C_2 + MA_{12} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} [1 \quad 0] = \begin{bmatrix} M_1 & 0 \\ M_2+1 & 0 \end{bmatrix} \tag{4-10}$$

which clearly shows F_2 does not have full rank for any choice of M. This corresponds to the fact that the system matrix (Ref 13)

$$\begin{bmatrix} \lambda I-A & B \\ C & 0 \end{bmatrix} \tag{4-11}$$

is rank deficient for $\lambda=0$ (transmission zero at the origin). Therefore, the system matrix may be rewritten

$$\begin{bmatrix} A & B \\ -C & 0 \end{bmatrix} \tag{4-12}$$

which, for a choice of q (pitch rate) as an output, does not have full rank. Therefore, the system is not fully controllable, which is a necessary condition for successful application of this theory. The same result is true for a selection of normal acceleration, A_N, as an output.

Case A-6

This case is identical with Case A-1 except that Σ is chosen to be diag $\{0.1,0.1\}$ rather than diag $\{1.0,1.0\}$. The resulting system matrices are identical to Case A-1 except K_o and K_1 are one-tenth that of the former case. If the gain in this case is chosen to be ten times that of any example from Case A-1, the closed-loop eigenvalues and all responses are identical, indicating no beneficial

effects from increasing or decreasing Σ by a constant factor. This result was found to be true in all cases.

System B - Short Period Approximation With Control Actuator Dynamics

The next set of test cases have control actuator dynamics modeled in the aircraft state equations. The model used is shown in Fig. 3-6, from which the matrices

$$A_{11} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -0.86939 & 43.233 \\ 0 & 0.99335 & -1.3411 \end{bmatrix} \quad A_{12} = \begin{bmatrix} 0 & 0 \\ -17.251 & -1.5766 \\ -0.16897 & -0.25183 \end{bmatrix} \tag{4-13}$$

$$A_{21} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad A_{22} = \begin{bmatrix} -20 & 0 \\ 0 & -20 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad B_2 = \begin{bmatrix} 20 & 0 \\ 0 & 20 \end{bmatrix} \tag{4-14}$$

are easily recognized. The output matrix for pitch pointing and vertical translation is given by

$$[C_1 \mid C_2] = \left[\begin{array}{ccc|cc} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \end{array}\right] \tag{4-15}$$

and that for direct lift control by

$$[C_1 \mid C_2] = \left[\begin{array}{ccc|cc} 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{array}\right] \tag{4-16}$$

In both cases C_2 is rank deficient and extra plant measurements are required.

In these test cases, three slow eigenvalues must be selected, thereby allowing complex as well as real eigenvalues to be chosen. As two vectors lie in the null space of $[\lambda_i I-A_{11}, A_{12}]$, $i = 1, 2, 3$, the selection of six eigenvector multipliers is required to form the three closed-loop eigenvectors.

Case B-1

In this case the chosen eigenvalues are $\{(-4\pm 3j); (-3+j0)\}$, and the output equations are those represented by equation (4-14). Appendix B discusses the selection of complex eigenvalues. In the cases where complex eigenvalues are selected, the last two of the four corresponding vectors lying in the null space are assigned zero multipliers to reduce the complexity of the problem. This approach still reproduces the requested eigenvalues.

Since the control surface deflections are now included as state equations, the control time histories are easy to evaluate. In the previous test cases without actuator dynamics, the control law has to be evaluated and integrated in order to obtain these histories. This is but one advantage to modeling control actuator dynamics.

Several choices of eigenvector multipliers are made in this case. First, multipliers {1, 10, 0, 0, 1, 1} are chosen and the resulting system matrices and eigenvalue movements are shown in Fig. 4-14 and Table 4-6, respectively. The corresponding control histories and time responses for g = 1000 are shown in Figs. 4-15 through 4-18 for command

$$M = \begin{bmatrix} 0.2871 & 0.0779 & 0.0914 \\ -0.1104 & -0.0882 & -0.2748 \end{bmatrix}$$

$$[F_1 \,\vdots\, F_2] = \left[\begin{array}{ccc|cc} 1 & 0.3102 & 3.2437 & -1.3587 & -0.1458 \\ 1 & -0.3067 & -4.4462 & 1.5686 & 0.2083 \end{array}\right]$$

$$F_2 B_2 = \begin{bmatrix} -21.1737 & -2.9157 \\ 31.3723 & 4.1666 \end{bmatrix}$$

$$\Sigma = \text{diag } \{1,1\}$$

$$K_o = \begin{bmatrix} -0.1916 & -0.1341 \\ 1.4425 & 1.2494 \end{bmatrix} = \frac{1}{2} K_1$$

Fig. 4-14. System Matrices for Case B-1, Multipliers {1,10,0,0,1,1}

Table 4-6. Eigenvalue Movements for Case B-1, Multipliers {1,10,0,0,1,1}

gain	Eigenvalues						
0	(0,0)	(0,0)	(0,0)	(5.45,0)	(-7.66,0)	(-20,0)	(-20,0)
1	(-0.32,1.03)	(-0.32,-1.03)	(0.015,0)	(5.36,0)	(-7.40,0)	(-20.66,0)	(-20.89,0)
5	(-1.39,1.79)	(-1.39,-1.79)	(0.08,0)	(4.85,0)	(-6.46,0)	(-23.53,0)	(-24.37,0)
10	(-2.52,1.85)	(-2.52,-1.85)	(0.18,0)	(4.13,0)	(-5.24,0)	(-27.61,0)	(-28.63,0)
16.8	(-4.12,2.25)	(-4.12,-2.25)	(0.36,0)	(3.17,0)	(-2.91.0)	(-34.1,0.06)	(-34.1,-0.06)
27.5	(-4.73,0.86)	(-4.73,-0.86)	(1.26,0.19)	(1.26,-0.19)	(-2.37,0)	(-43.9,0.95)	(-43.9,-0.95)
50	(-5.05,3.07)	(-5.05,-3.07)	(0.54,1.42)	(0.54,-1.42)	(-2.17,0)	(-65.5,1.70)	(-65.5,-1.7)
100	(-5.06,30.2)	(-5.06,-3.02)	(-0.31,1.75)	(-0.31,-1.75)	(-2.08,0)	(-114.7,2.32)	(-114.7,-2.32)
500	(-4.50,2.82)	(-4.50,-2.82)	(-1.75,1.39)	(-1.75,-1.39)	(-2.01,0)	(-513.8,2.92)	(-513.8,-2.92)
1000	(-4.27,2.85)	(-4.27,-2.85)	(-2.11,1.05)	(-2.11,-1.05)	(-2.01,0)	(-1014,3.0)	(-1014,-3.0)
6000	(-4.04,2.96)	(-4.04,-2.96)	(-2.44,0.05)	(-2.44,-0.05)	(-2.00,0)	(-6013.6,3.07)	(-6013.6,-3.07)
10,000	(-4.02,2.98)	(-4.02,-2.98)	(-2.76,0)	(-2.15,0)	(-2.00,0)	(-10014,3.1)	(-10014,-3.1)
∞	(-4,3)	(-4,-3)	(-3,0)	(-2,0)	(-2,0)	(-∞,0)	(-∞,0)

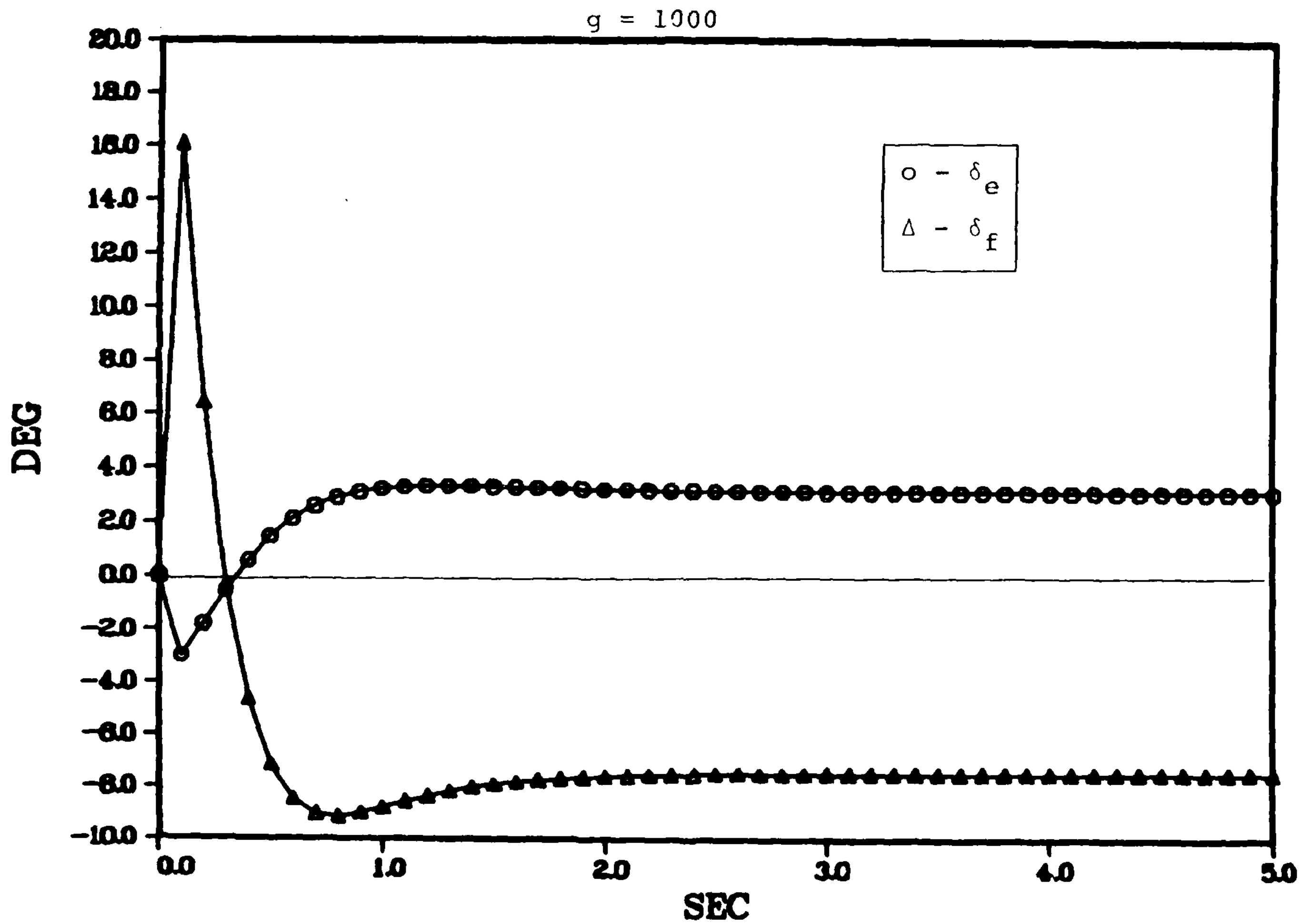

Fig. 4-15. Control Histories for Case B-1, Pitch Pointing Maneuver with Multipliers {1,10,0,0,1,1}

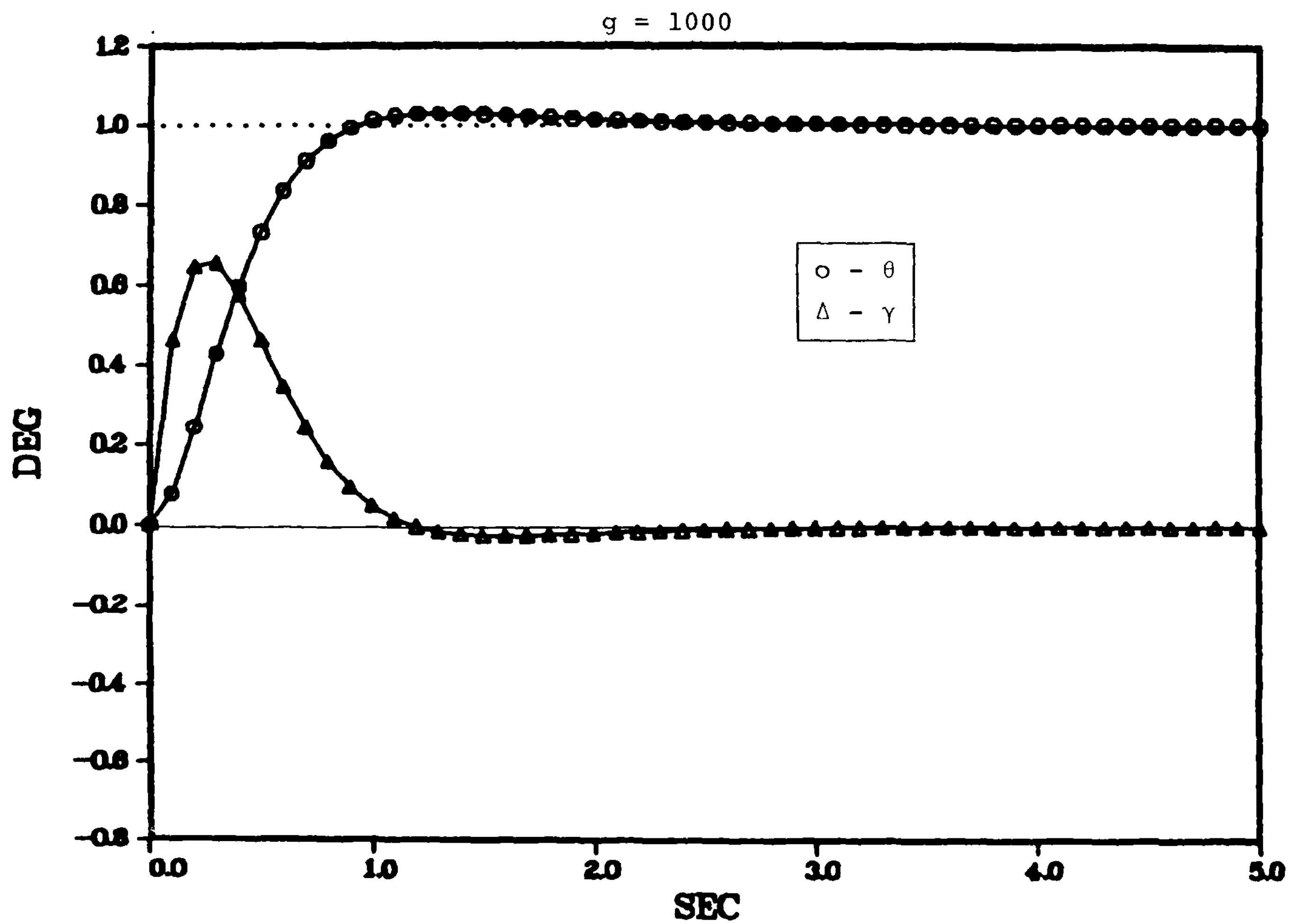

Fig. 4-16. Time Responses for Case B-1, Pitch Pointing Maneuver with Multipliers {1,10,0,0,1,1}

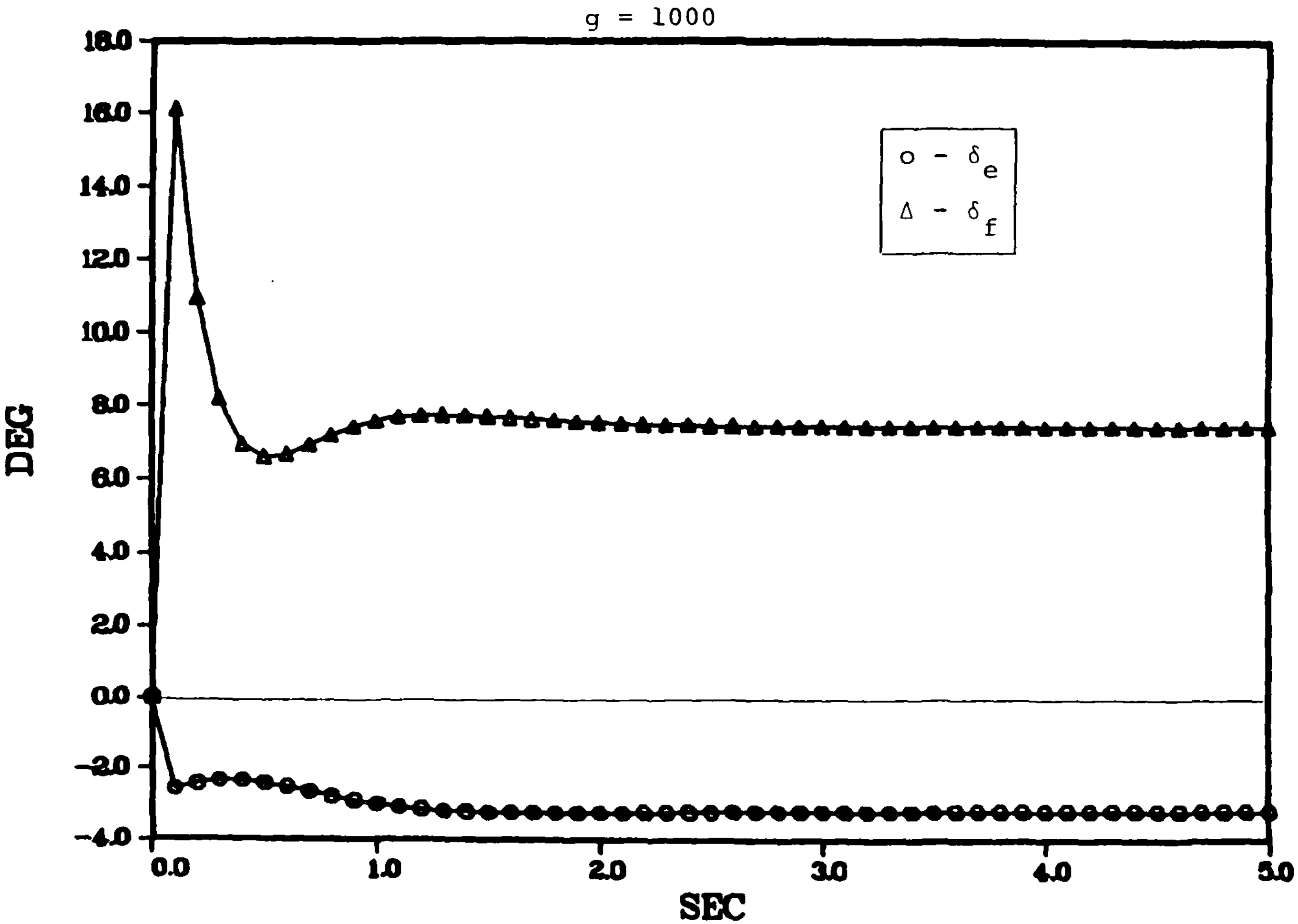

Fig. 4-17. Control Histories for Case B-1, Vertical Translation Maneuver with Multipliers {1,10,0,0,1,1}

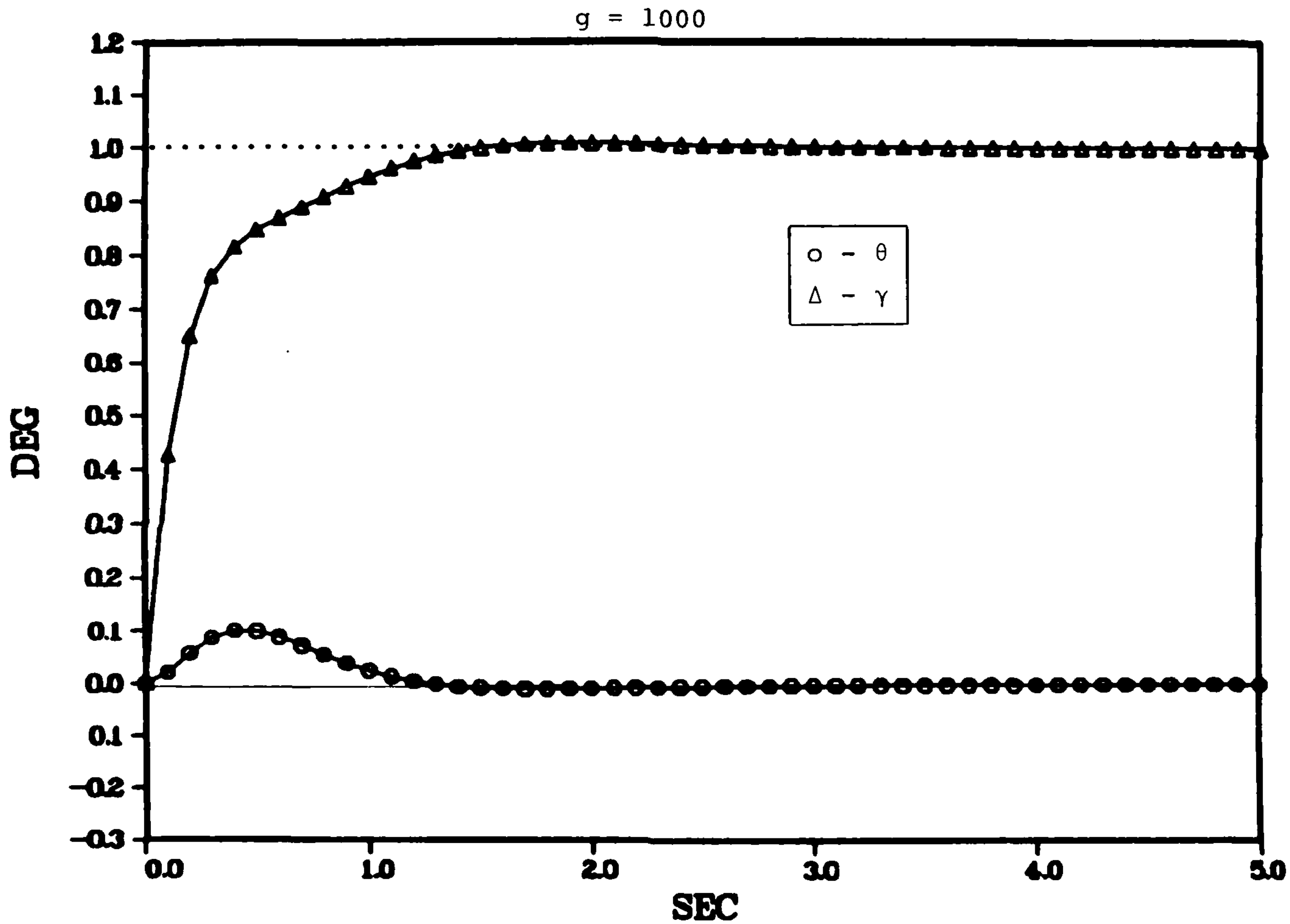

Fig. 4-18. Time Responses for Case B-1, Vertical Translation Maneuver with Multipliers {1,10,0,0,1,1}

vectors $\underline{v} = [1\ 0]^T$deg and $\underline{v} = [0\ 1]^T$deg. These results show the control surface deflections to be very abrupt in the first second and show the outputs to be significantly interactive (especially in the pitch pointing mode).

Next the multipliers are changed to {1, 1, 0, 0, 1, 0} with corresponding results for g = 1000 shown in Fig. 4-19, Table 4-7, and Figs. 4-20 through 4-23. These results indicate two major trends. First, the magnitude of the elements in the F_2B_2 matrix is greatly reduced from that of the previous case. Second, the control surface deflections are not as abrupt in this case. The time responses in this case are greatly improved over the previous one.

Changing the multipliers slightly to {1, 1, 0, 0, 1, 1} produces even better responses. Results in this case are shown in Fig. 4-24, Table 4-8, and Figs. 4-25 through 4-28 for g = 1000. The magnitude of the elements of the F_2B_2 matrix show little change over the previous case, but the control deflections are slightly less abrupt. Consequently, the time response is made much more non-interactive.

Case B-2

The output equations given by equation (4-15) along with the choices of {$(-4\pm3j)$; $(-3+j0)$} for the slow eigenvalues are used in this case. Numerous choices of eigenvector multipliers were made, with two examples shown here. In the first example, the choice of {1,0,0,0,1,0}

$$M = \begin{bmatrix} 0.1571 & 0.0376 & 0.1488 \\ 0.3663 & 0.0019 & -0.3474 \end{bmatrix}$$

$$[F_1 \,\vdots\, F_2] = \left[\begin{array}{ccc:cc} 1 & 0.2726 & 1.4249 & -0.6734 & -0.0967 \\ 1 & 0.0196 & -0.4529 & 0.0263 & 0.0845 \end{array}\right]$$

$$F_2B_2 = \begin{bmatrix} -13.4671 & -1.9343 \\ 0.5258 & 1.6905 \end{bmatrix}$$

$$\Sigma = \text{diag}\ \{1,1\}$$

$$K_o = \begin{bmatrix} -0.0777 & -0.0889 \\ 0.0242 & 0.6192 \end{bmatrix} = \frac{1}{2}\ K_1$$

Fig. 4-19. System Matrices for Case B-1, Multipliers {1,1,0,0,1,0}

Table 4-7. Eigenvalue Movements for Case B-1, Multipliers {1,1,0,0,1,0}

gain, g	Eigenvalues						
0	(0,0)	(0,0)	(0,0)	(5.45,0)	(-7.66,0)	(-20,0)	(-20,0)
1	(-0.10,0.38)	(-0.10,-0.38)	(0.12,0)	(4.98,0)	(-7.57,0)	(-20.69,0)	(-20.85,0)
5	(-0.42,0.73)	(-0.42,-0.73)	(0.86,0)	(2.96,0)	(-7.26,0)	(-23.58,0)	(-24.35,0)
6.2	(-0.50,0.78)	(-0.50,-0.78)	(1.74,0.09)	(1.74,-0.09)	(-7.13,0)	(-24.49,0)	(-25.42,0)
10	(-0.71,0.86)	(-0.71,-0.86)	(1.25,1.67)	(1.25,-1.67)	(-6.96,0)	(-27.45,0)	(-28.87,0)
25	(-1.20,0.89)	(-1.20,-0.89)	(-0.08,2.76)	(-0.08,-2.76)	(-6.34,0)	(-40.26,0)	(-43.04,0)
50	(-1.56,0.76)	(-1.56,-0.76)	(-1.21,2.97)	(-1.21,-2.97)	(-5.69,0)	(-63.56,0)	(-67.44,0)
100	(-1.81,0.57)	(-1.81,-0.57)	(-2.19,2.87)	(-2.19,-2.87)	(-4.89,0)	(-112.33,0)	(-117.0,0)
250	(-1.98,0.31)	(-1.98,-0.31)	(-3.20,2.77)	(-3.20,-2.77)	(-3.75,0)	(-261.44,0)	(-266.68,0)
500	(-2.01,0.17)	(-2.01,-0.17)	(-3.62,2.85)	(-3.62,-2.85)	(-3.27,0)	(-511.1,0)	(-516.6,0)
1000	(-2.02,0.09)	(-2.02,-0.09)	(-3.82,2.92)	(-3.82,-2.92)	(-3.10,0)	(-1010.9,0)	(-1016.5,0)
5000	(-2.01,0.02)	(-2.01,-0.02)	(-3.96,2.98)	(-3.96, 2.98)	(-3.01,0)	(-5010.8,0)	(-5016.4,0)
∞	(-2,0)	(-2,0)	(-4,3)	(-4,-3)	(-3,0)	(-∞,0)	(-∞,0)

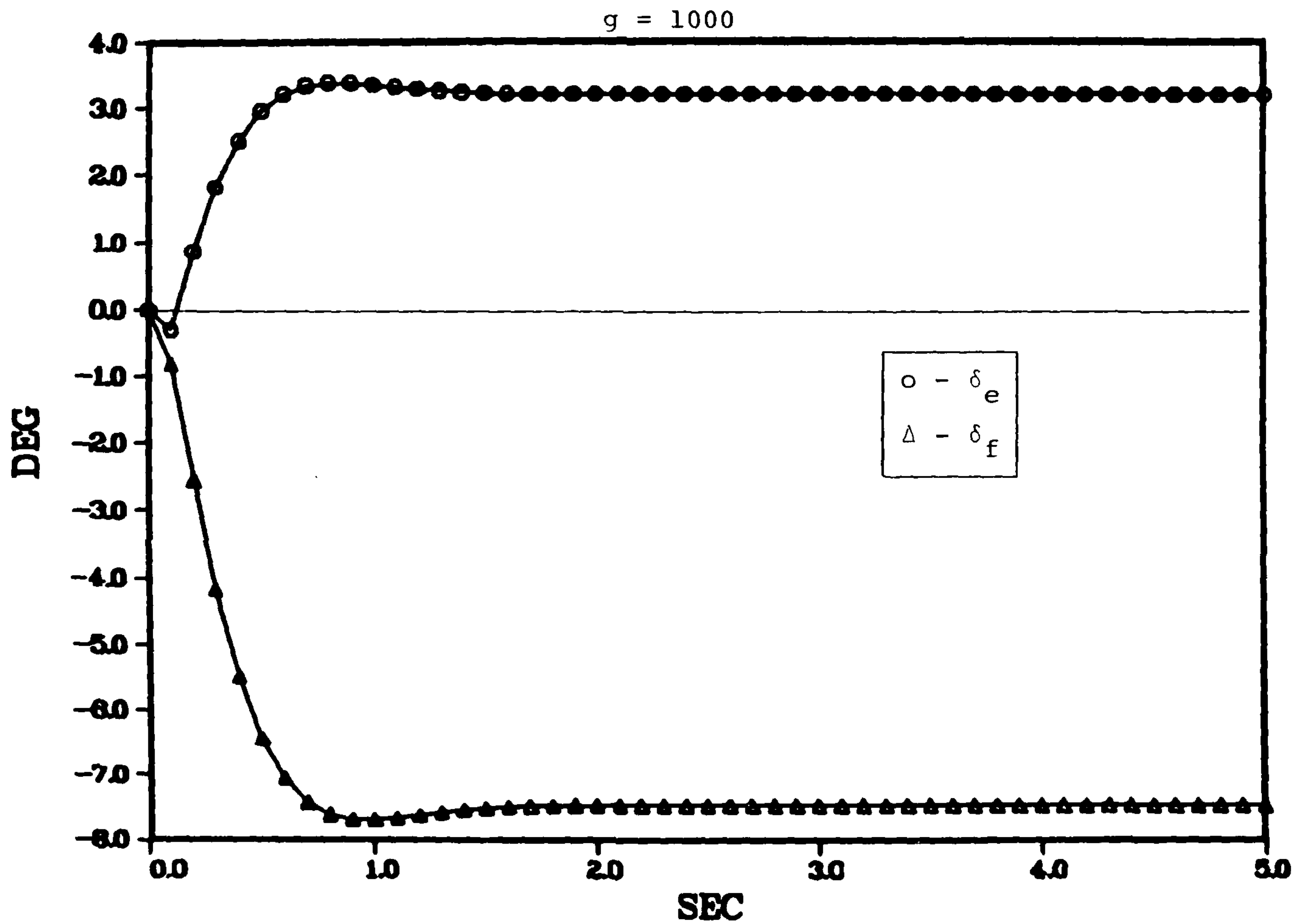

Fig. 4-20. Control Histories for Case B-1, Pitch Pointing Maneuver with Multipliers {1,1,0,0,1,0}

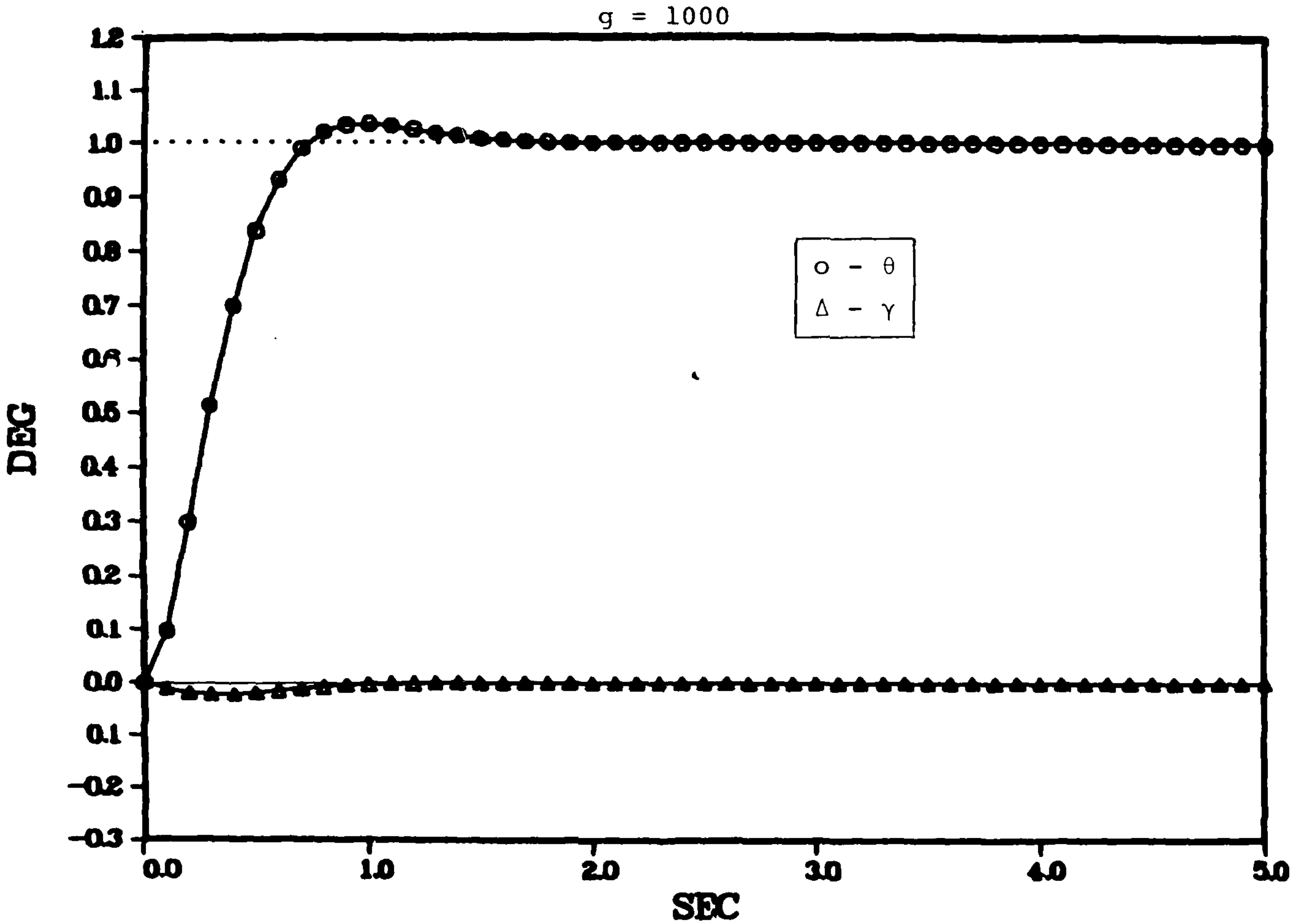

Fig. 4-21. Time Responses for Case B-1, Pitch Pointing Maneuver with Multipliers {1,1,0,0,1,0}

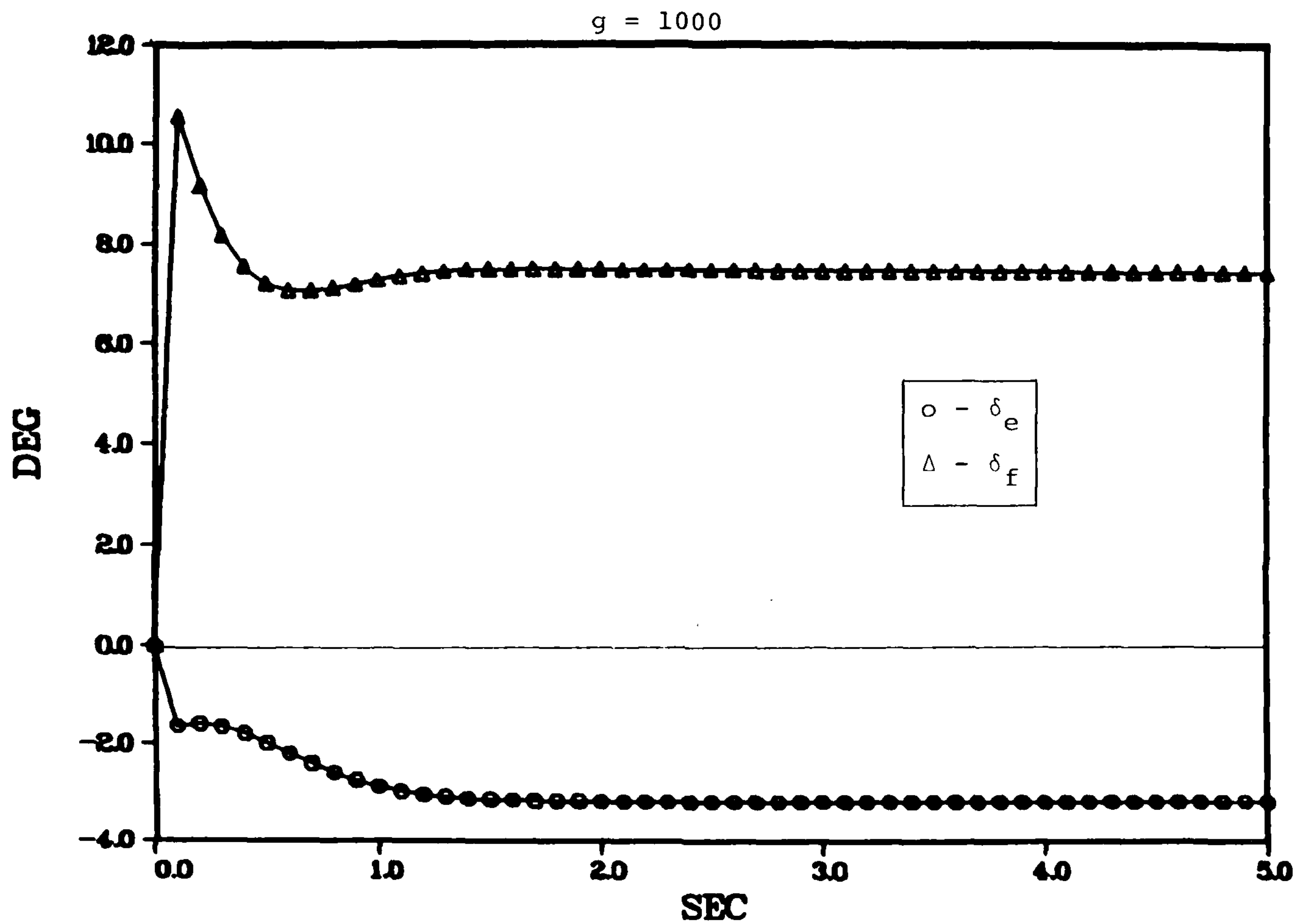

Fig. 4-22. Control Histories for Case B-1, Vertical Translation Maneuver with Multipliers {1,1,0,0,1,0}

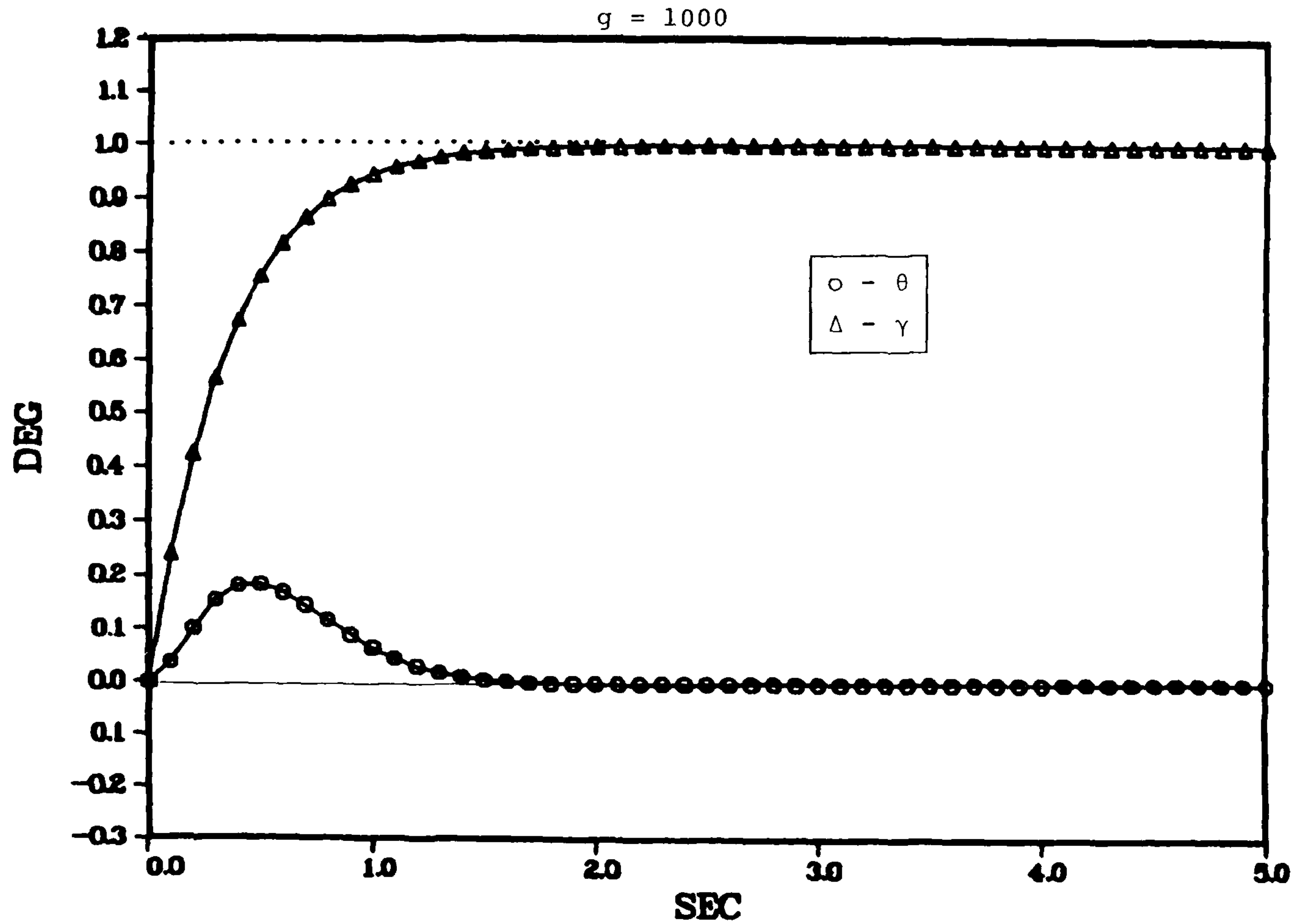

Fig. 4-23. Time Responses for Case B-1, Vertical Translation Maneuver with Multipliers {1,1,0,0,1,0}

$$M = \begin{bmatrix} 0.2476 & 0.0389 & 0.0662 \\ 0.3577 & 0.0018 & -0.3396 \end{bmatrix}$$

$$[F_1 \vdots F_2] = \left[\begin{array}{ccc:cc} 1 & 0.2795 & 1.594 & -0.6826 & -0.078 \\ 1 & 0.0189 & -0.4689 & 0.0272 & 0.0828 \end{array}\right]$$

$$F_2B_2 = \begin{bmatrix} -13.6524 & -1.5605 \\ 0.5434 & 1.6552 \end{bmatrix}$$

$$\Sigma = \text{diag}\ \{1,1\}$$

$$K_o = \begin{bmatrix} -0.0761 & -0.0718 \\ 0.025 & 0.6277 \end{bmatrix} = \frac{1}{2} K_1$$

Fig. 4-24. System Matrices for Case B-1, Multipliers {1,1,0,0,1,1}

Table 4-8. Eigenvalue Movements for Case B-1, Multipliers {1,1,0,0,1,1}

gain, g	Eigenvalues						
0	(0,0)	(0,0)	(0,0)	(5.45,0)	(-7.66,0)	(-20,0)	(-20,0)
1	(-0.10,0.42)	(-0.10,-0.42)	(0.099,0)	(5.00,0)	(-7.57,0)	(-20.69,0)	(-20.85,0)
5	(-0.42,0.80)	(-0.42,-0.80)	(0.71,0)	(3.10,0)	(-7.25,0)	(-23.59,0)	(-24.34,0)
6.65	(-0.53,0.87)	(-0.53,-0.87)	(1.67,0.11)	(1.67,-0.11)	(-7.13,0)	(-24.84,0)	(-25.81,0)
10	(-0.72,0.95)	(-0.72,-0.95)	(1.24,1.52)	(1.24,-1.52)	(-6.93,0)	(-27.47,0)	(-28.85,0)
25	(-1.24,0.97)	(-1.24,-0.97)	(-0.08,2.63)	(-0.08,-2.63)	(-6.28,0)	(-40.31,0)	(-42.98,0)
50	(-1.60,0.84)	(-1.60,-0.84)	(-1.20,2.85)	(-1.20,-2.85)	(-5.61,0)	(-63.65,0)	(-67.35,0)
100	(-1.86,0.62)	(-1.86,-0.62)	(-2.20,2.76)	(-2.20,-2.76)	(-4.76,0)	(-112.44,0)	(-116.89,0)
250	(-2.03,0.34)	(-2.03,-0.34)	(-3.24,2.72)	(3.24,-2.72)	(-3.56,0)	(-261.56,0)	(-266.55,0)
500	(-2.05,0.18)	(-2.05,-0.18)	(-3.65,2.84)	(-3.65,-2.84)	(-3.15,0)	(-511.25,0)	(-516.43,0)
1000	(-2.04,0.09)	(-2.04,-0.09)	(-3.83,2.92)	(-3.83,-2.92)	(-3.03,0)	(-1011.1,0)	(-1016.4,0)
∞	(-2,0)	(-2,0)	(-4,3)	(-4,-3)	(-3,0)	(-∞,0)	(-∞,0)

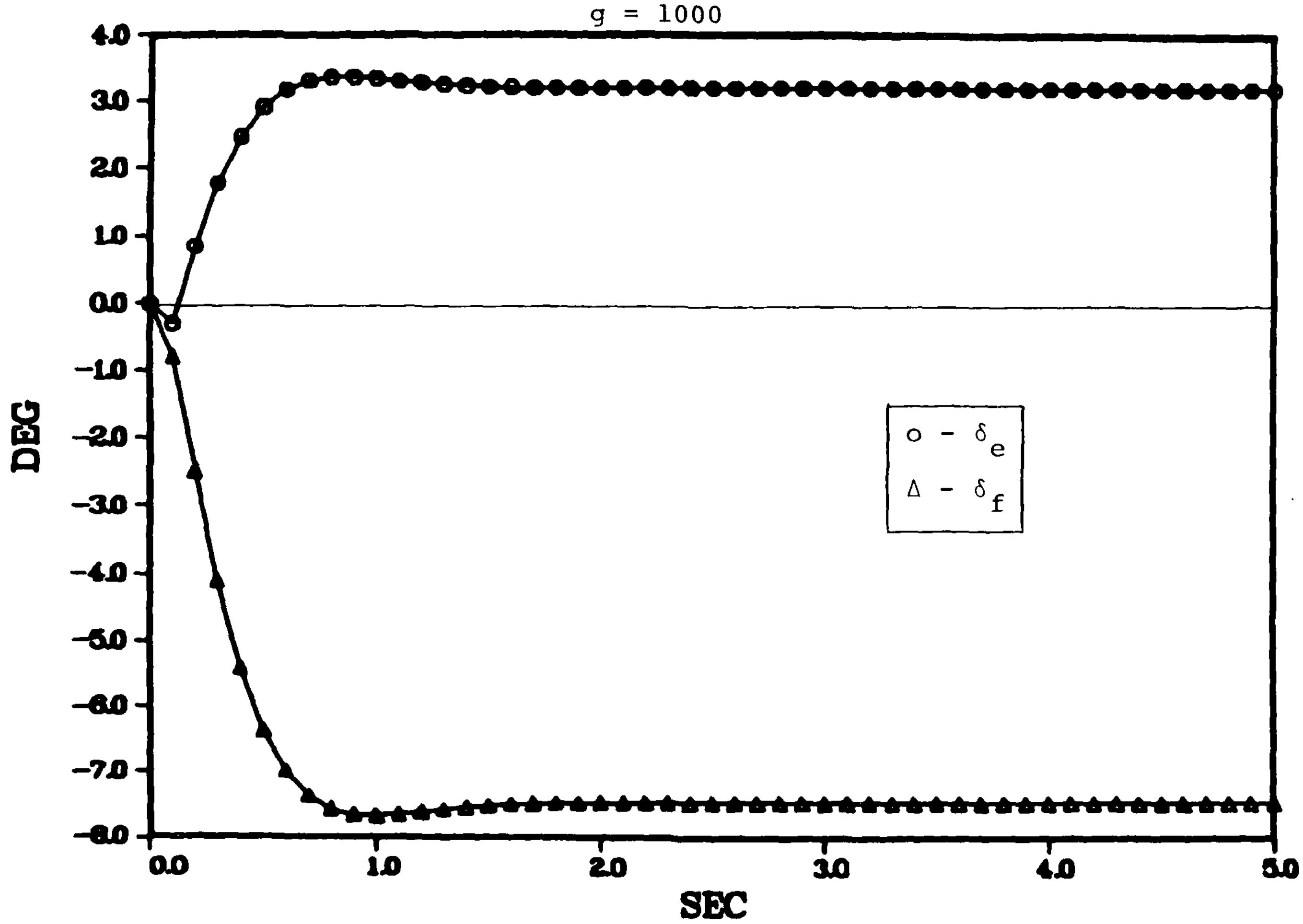

Fig. 4-25. Control Histories for Case B-1, Pitch Pointing Maneuver with Multipliers {1,1,0,0,1,1}

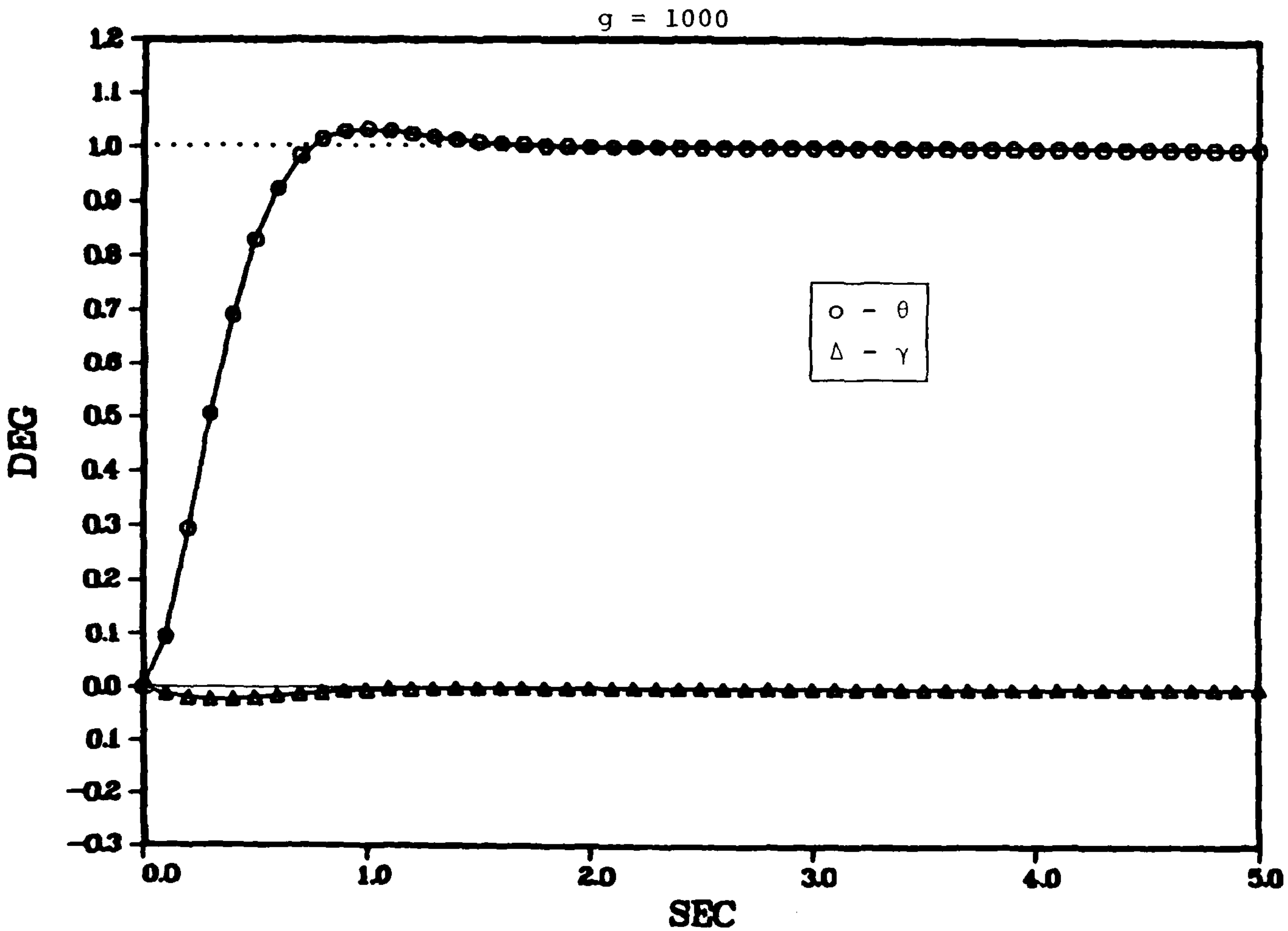

Fig. 4-26. Time Responses for Case B-1, Pitch Pointing Maneuver with Multipliers {1,1,0,0,1,1}

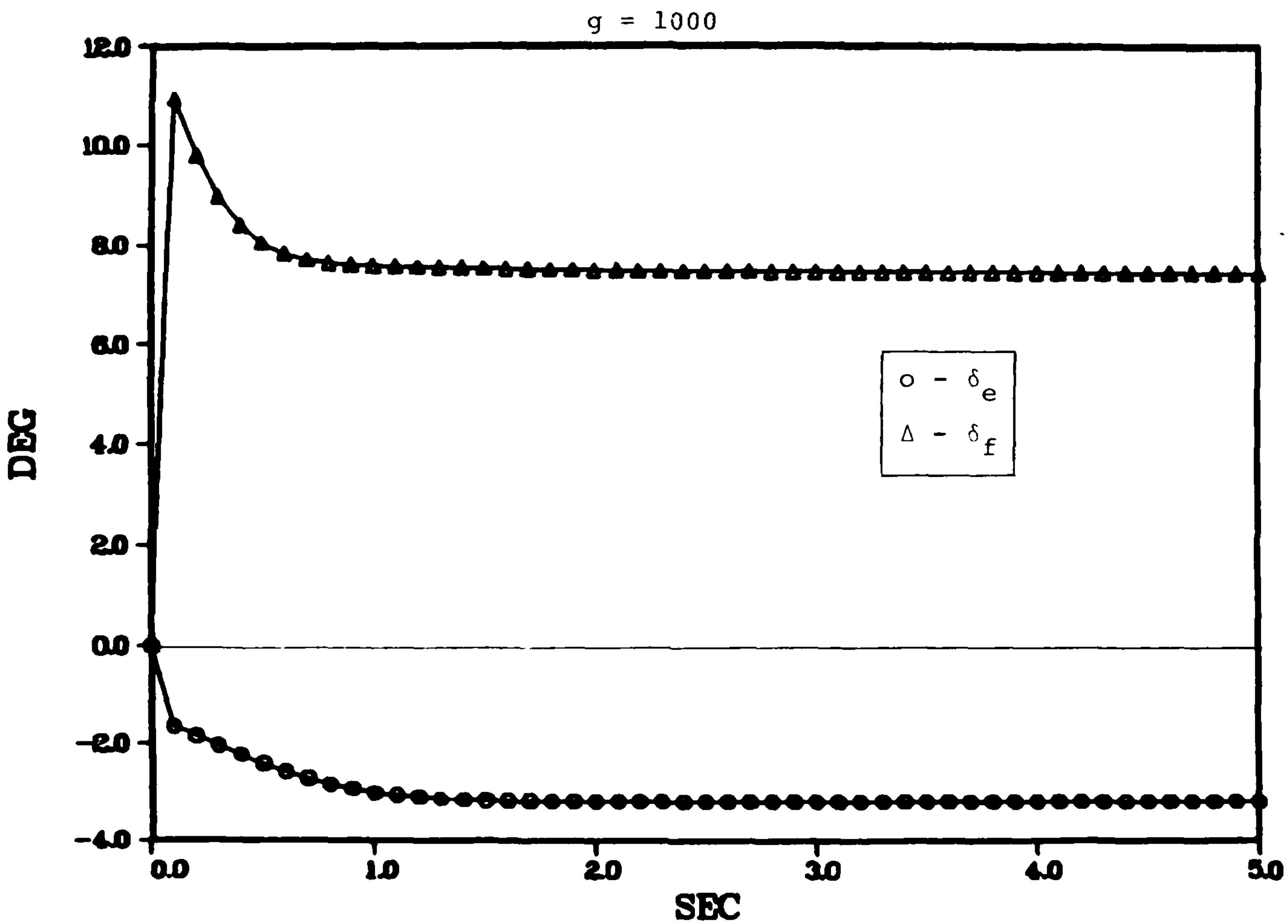

Fig. 4-27. Control Histories for Case B-1, Vertical Translation Maneuver with Multipliers {1,1,0,0,1,1}

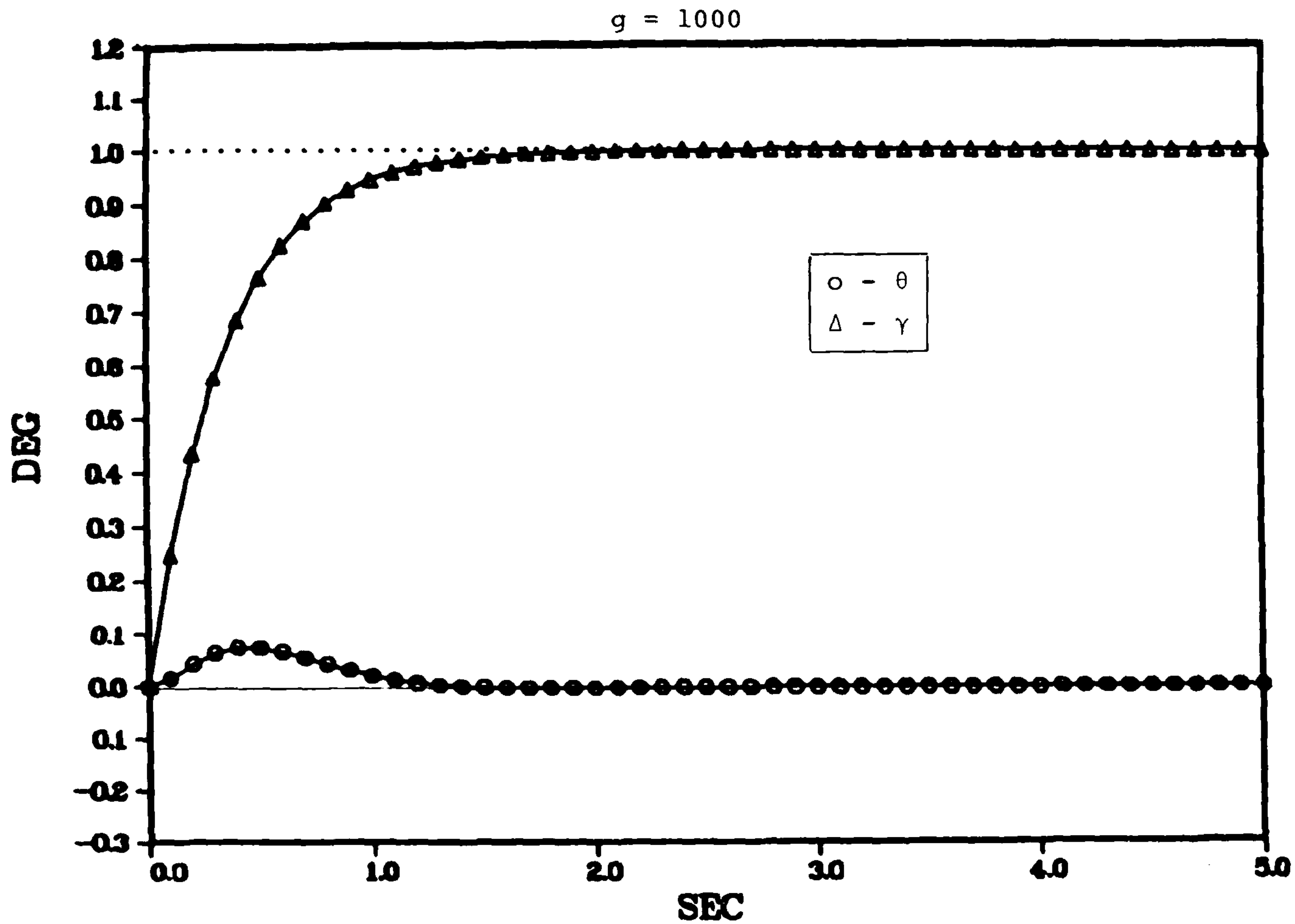

Fig. 4-28. Time Responses for Case B-1, Vertical Translation Maneuver with Multipliers {1,1,0,0,1,1}

for multipliers results in the matrices, eigenvalues, and time histories shown in Fig. 4-29, Table 4-9, and Figs. 4-30 and 4-31 for g = 1000. An interesting result here is that the control surface deflections (Fig. 4-30) both "jumped" to a maximum value and then quickly returned to equilibrium. This implies some type of new trim condition when effecting the direct lift control maneuver, which is unrealistic. In this case the short period approximation is physically invalid since the direct lift control maneuver effects a forward speed change, and u is assumed to be zero in a short period approximation. The time response in Fig. 4-31 shows satisfactory achievement of the final commanded values but with a great deal of output interaction in the first two seconds.

In an attempt to reduce this interaction the eigenvector multipliers are changed, with the best case found to be {1,1,0,0,1,1}. This is the same as the best case in the previous test case and produces the same eigenvalue movements as shown in Fig. 4-24. The system matrices, control histories, and time responses for g = 1000 are shown in Figs. 4-32 through 4-34. Fig. 4-34 (the time responses) still indicates interaction between the outputs but it is much less than in the first example.

<u>Case B-3</u>

In this test case the chosen slow eigenvalues are {(-4+j0); (-4+j0); (-2+j0)} with the output equations given by equation (4-15). As explained in Appendix B, generalized

$$M = \begin{bmatrix} 0.7431 & 0.0361 & -0.4886 \\ -0.7937 & -0.0247 & 0.7039 \end{bmatrix}$$

$$[F_1 \,\vdots\, F_2] = \left[\begin{array}{ccc:cc} 1 & 0.2264 & 1.2158 & -0.5401 & 0.0661 \\ 0 & -0.073 & -1.0125 & 0.3074 & -0.1383 \end{array}\right]$$

$$F_2 B_2 = \begin{bmatrix} -10.8022 & 1.3288 \\ 6.1476 & -2.7662 \end{bmatrix}$$

$$\Sigma = \text{diag}\ \{1,1\}$$

$$K_o = \begin{bmatrix} -0.1272 & -0.0608 \\ -0.2827 & -0.4967 \end{bmatrix} = \frac{1}{2} K_1$$

Fig. 4-29. System Matrices for Case B-2, Multipliers {1,0,0,0,1,0}

Table 4-9. Eigenvalue Movements for Case B-2, Multipliers {1,0,0,0,1,0}

gain, g	Eigenvalues						
0	(0,0)	(0,0)	(0,0)	(5.45,0)	(-7.66,0)	(-20,0)	(-20,0)
1	(-0.05,0)	(-0.50)	(0.82,0)	(4.69,0)	(-7.60,0)	(-20.61,0)	(-20.96,0)
2.56	(-0.11,0)	(-0.73,0)	(2.54,0.08)	(2.54,-0.08)	(-7.51,0)	(-21.60,0)	(-22.46,0)
5	(-0.21,0)	(-0.93,0)	(2.17,2.12)	(2.17,-2.12)	(-7.38,0)	(-23.20,0)	(-24.83,0)
10	(-0.35,0)	(-1.17,0)	(1.46,3.26)	(1.46,-3.26)	(-7.17,0)	(-26.73,0)	(-29.71,0)
25	(-0.63,0)	(-1.49,0)	(-0.04,4.15)	(-0.04,-4.15)	(-6.72,0)	(-38.81,0)	(-44.49,0)
50	(-0.88,0)	(-1.68,0)	(-1.25,4.18)	(-1.25,-4.18)	(-6.23,0)	(-61.57,0)	(-69.34,0)
100	(-1.13,0)	(-1.82,0)	(-2.20,3.87)	(-2.20,-3.87)	(-5.65,0)	(-110.0,0)	(-119.2,0)
250	(-1.42,0)	(-1.92,0)	(-3.02,3.39)	(-3.02,-3.39)	(-4.78,0)	(-258.9,0)	(-269.1,0)
500	(-1.60,0)	(-1.96,0)	(-3.43,3.16)	(-3.43,-3.16)	(-4.15,0)	(-508.5,0)	(-519.1,0)
1000	(-1.75,0)	(-1.98,0)	(-3.70,3.06)	(-3.70,-3.06)	(-3.67,0)	(-1008.3,0)	(-1019.1,0)
5000	(-1.93,0)	(-2.00,0)	(-3.94,3.01)	(-3.94,-3.01)	(-3.16,0)	(-5008.2,0)	(-5019.1,0)
∞	(-2,0)	(-2,0)	(-4,3)	(-4,-3)	(-3,0)	(- ∞,0)	(- ∞,0)

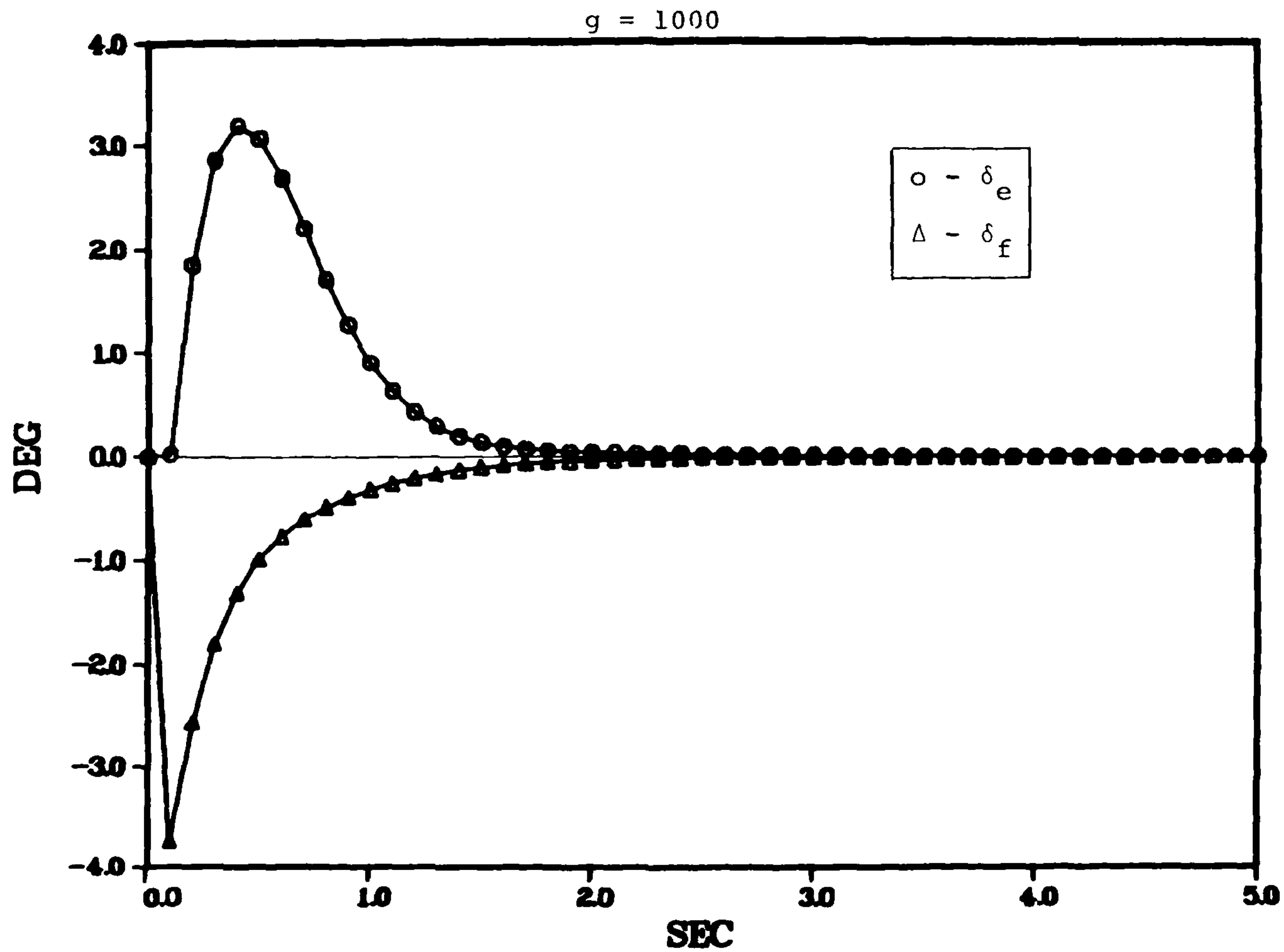

Fig. 4-30. Control Histories for Case B-2, Direct Lift Control Maneuver with Multipliers {1,0,0,0,1,0}

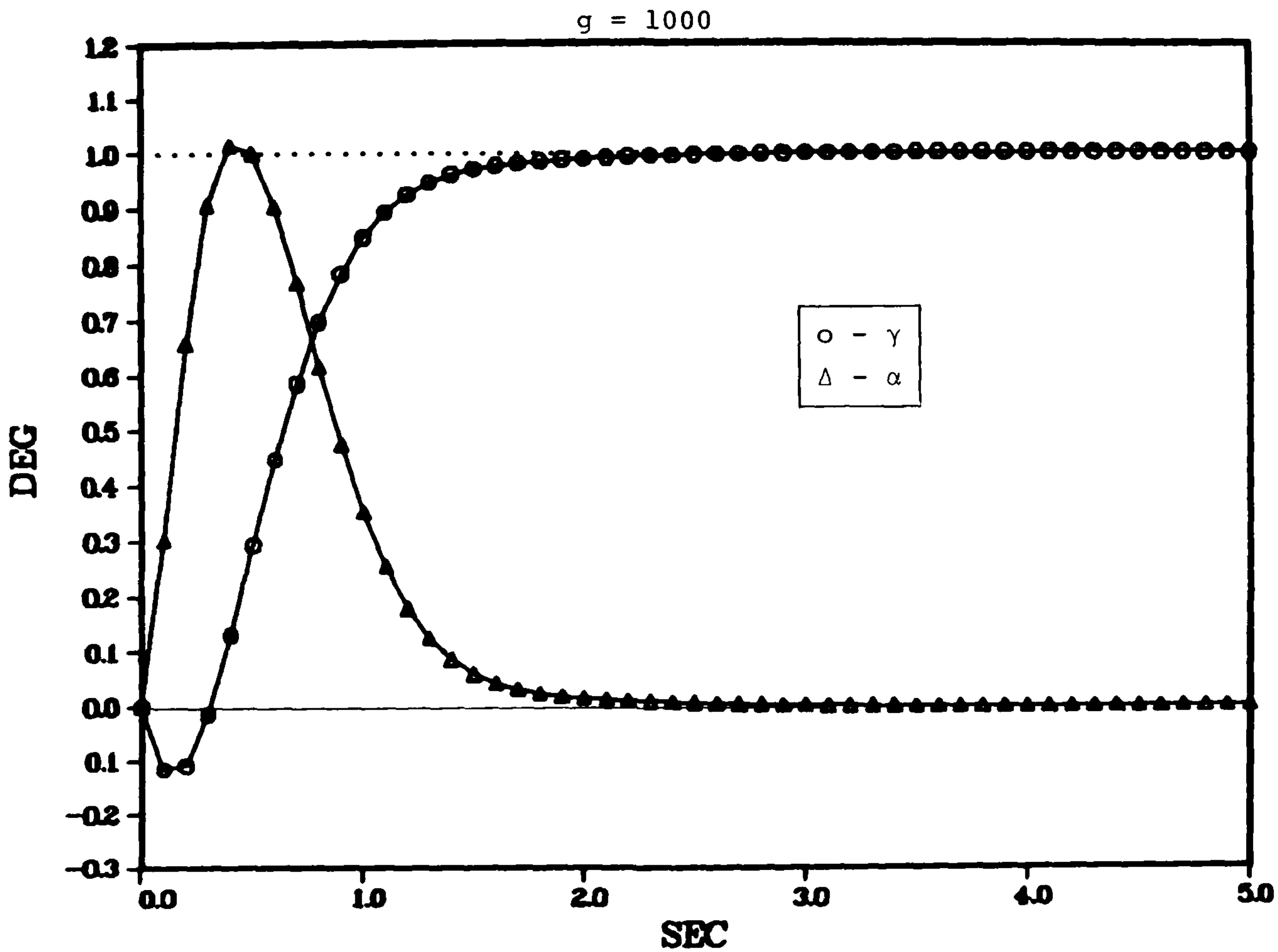

Fig. 4-31. Time Responses for Case B-2, Direct Lift Control Maneuver with Multipliers {1,0,0,0,1,0}

$$M = \begin{bmatrix} 0.3577 & 0.0018 & -0.3396 \\ -0.1101 & 0.0372 & -0.4057 \end{bmatrix}$$

$$[F_1 \mid F_2] = \left[\begin{array}{ccc|cc} 1 & 0.0189 & -0.4689 & 0.0272 & 0.0828 \\ 0 & 0.2606 & 2.0629 & -0.7098 & -0.1608 \end{array}\right]$$

$$F_2 B_2 = \begin{bmatrix} 0.5434 & 1.6552 \\ -14.1958 & -3.2157 \end{bmatrix}$$

$$\Sigma = \text{diag}\ \{1,1\}$$

$$K_o = \begin{bmatrix} -0.1479 & -0.0761 \\ 0.6527 & 0.0250 \end{bmatrix} = \frac{1}{2} K_1$$

Fig. 4-32. System Matrices for Case B-2, Multipliers {1,1,0,0,1,1}

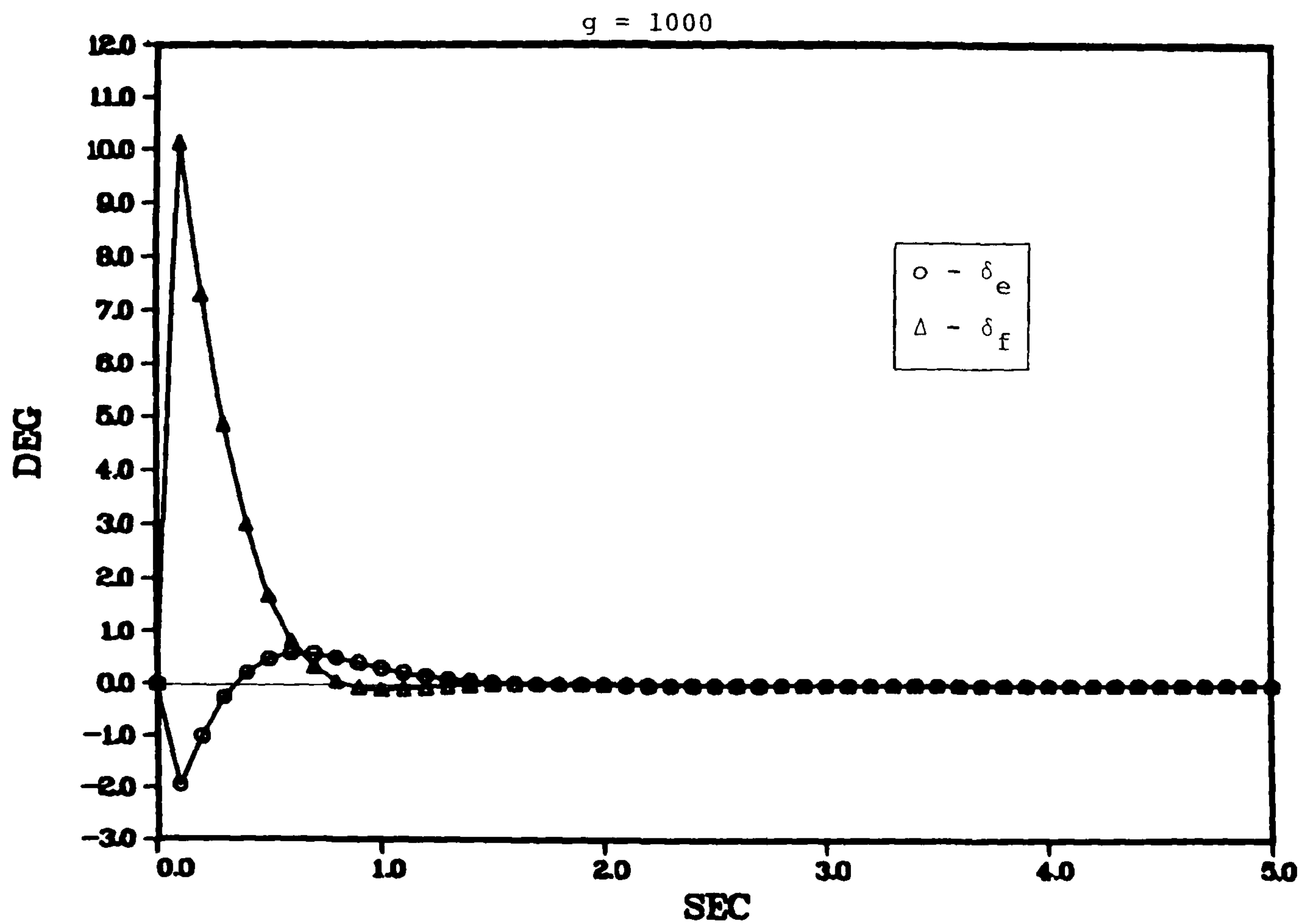

Fig. 4-33. Control Histories for Case B-2, Direct Lift Control Maneuver with Multipliers {1,1,0,0,1,1}

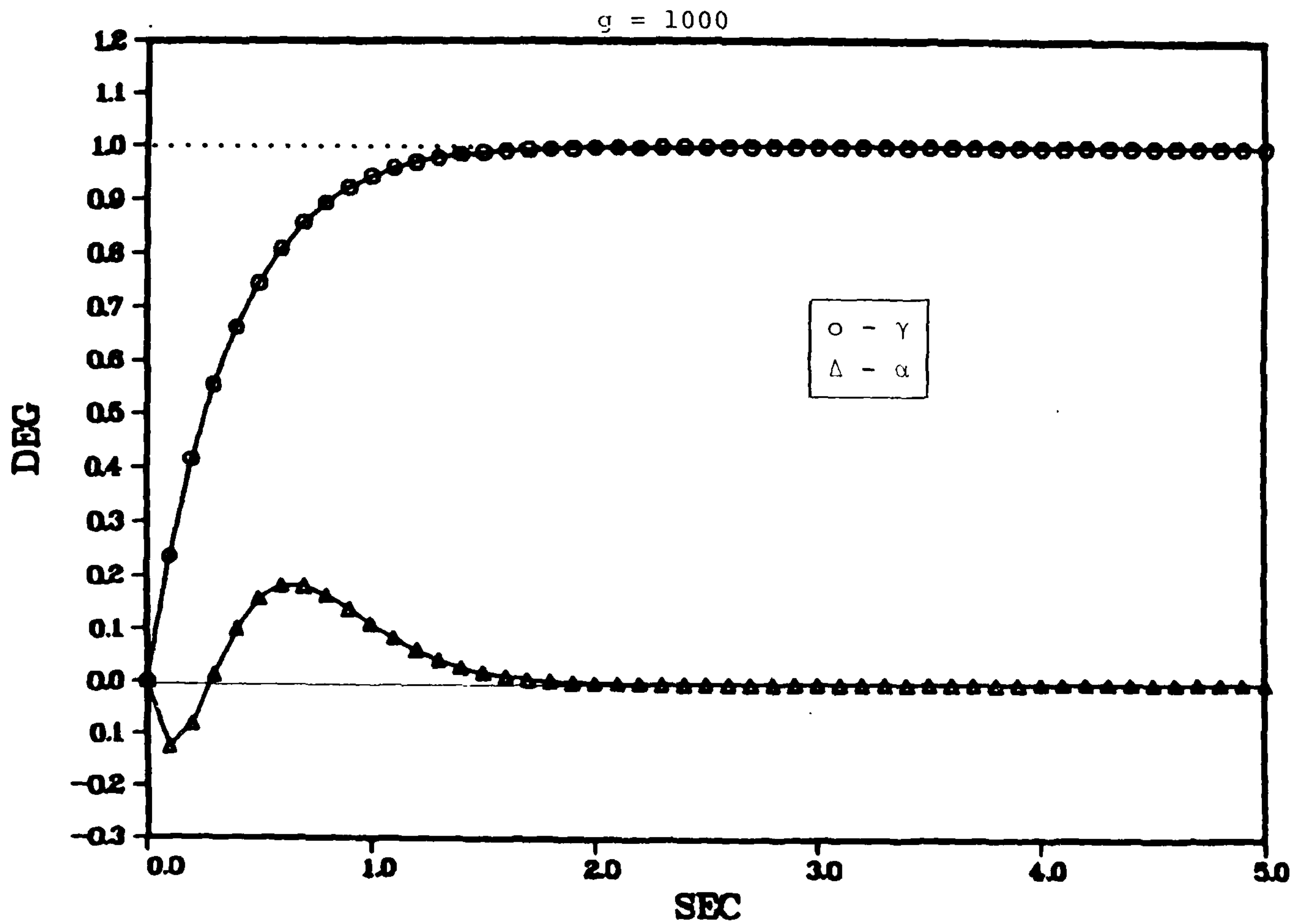

Fig. 4-34. Time Responses for Case B-2, Direct Lift Control Maneuver with Multipliers {1,1,0,0,1,1}

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USE OF ENTIRE EIGENSTRUCTURE ASSIGNMENT WITH HIGH-GAIN ERROR-AC--ETC(U)
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eigenvectors are not generated so that in order to produce repeated eigenvalues (and have $[\zeta_1, \zeta_2, \ldots \zeta_n]$ be invertible) a multiple of one null space vector is used for one eigenvalue a multiple of the other vector is used to produce the second eigenvalue. Testing shows that the results obtained are independent of the multiple of each vector used. In other words, the same results are obtained for a multiplier choice of [a,0,0,b,c,d] where a and b are different for each case but c and d are the same (the first four multipliers correspond to the repeated eigenvalues). Choosing {1,0,0,1,1,0} for the multipliers yields the results shown in Fig. 4-35, Table 4-10, and Figs. 4-36 through 4-43. To show the effect of increasing the gain parameter, g, Figs. 4-36 and 4-37 show the control and output histories, respectively, for g = 250 with command vector $\underline{v} = [1\ 0]^T$deg. Figs. 4-38 and 4-39 show the same results for g = 1000. It is easily seen that the response of $y_1 = \theta$ becomes tighter with increasing gain while the amount of interaction (y_2 response) changes very little. The same results, for g = 250 and g = 1000 but with $\underline{v} = [0\ 1]^T$deg, are shown in Figs. 4-40 through 4-43. Here the response of $y_2 = \gamma$ remains roughly the same while the amount of interaction is reduced significantly. These results show that the output y_2 always includes a significant amount, independent of the gain, of one or several of the modes defined by Z_2 (those which are selected as transmission zeros).

$$M = \begin{bmatrix} 0.75 & 0.125 & 0 \\ 0.3181 & 0.017 & -0.25 \end{bmatrix}$$

$$[F_1 \mid F_2] = \left[\begin{array}{ccc|cc} 1 & 0.6413 & 5.4041 & -2.1564 & -0.1971 \\ 1 & 0.0549 & 0.0708 & -0.2513 & 0.0361 \end{array}\right]$$

$$F_2B_2 = \begin{bmatrix} -43.1275 & -3.9415 \\ -5.025 & 0.7227 \end{bmatrix}$$

$$\Sigma = \text{diag } \{1,1\}$$

$$K_o = \begin{bmatrix} -0.0142 & -0.0773 \\ -0.0986 & 0.8461 \end{bmatrix} = \frac{1}{2} K_1$$

Fig. 4-35. System Matrices for Case B-3, Multipliers {1,0,0,1,1,0}

Table 4-10. Eigenvalue Movements for Case B-3, Multipliers {1,0,0,1,1,0}

gain,g	Eigenvalues					
0	(0,0)	(0,0)	(0,0)	(5.45,0)	(-7.66,0)	(-20,0)	(-20,0)
1	(-0.11,0.56)	(-0.11,-0.56)	(0.02,0)	(5.12,0)	(-7.56,0)	(-20.67,0)	(-20.91,0)
5	(-0.51,1.08)	(-0.51,-1.08)	(0.14,0)	(3.93,0)	(-7.22,0)	(-23.5,0)	(-24.55,0)
10	(-0.89,1.31)	(-0.89,-1.31)	(0.35,0)	(2.67,0)	(-6.92,0)	(-27.35,0)	(-29.17,0)
15.05	(-1.19,1.38)	(-1.19,-1.38)	(1.10,0.08)	(1.10,-0.08)	(-6.69,0)	(-31.52,0)	(-33.92,0)
50	(-2.13,1.14)	(-2.13,-1.14)	(-0.22,1.45)	(-0.22,-1.45)	(-5.88,0)	(-63.78,0)	(-67.85,0)
100	(-2.51,0.70)	(-2.51,-0.70)	(-0.82,1.40)	(-0.82,-1.40)	(-5.41,0)	(-112.7,0)	(-117.4,0)
165	(-2.68,0.08)	(-2.68,-0.08)	(-1.15,1.26)	(-1.15,-1.26)	(-5.10,0)	(-177.3,0)	(-182.2,0)
250	(-3.33,0)	(-2.25,0)	(-1.36,1.13)	(-1.36,-1.13)	(-4.87,0)	(-262.0,0)	(-267.1,0)
500	(-3.69,0)	(-2.10,0)	(-1.62,0.90)	(-1.62,-0.90)	(-4.56,0)	(-511.7,0)	(-516.9,0)
1000	(-3.85,0)	(-2.05,0)	(-1.78,0.70)	(-1.78,-0.70)	(-4.34,0)	(-1011.6,0)	(-1016.8,0)
5000	(-2.01,0)	(-3.97,0)	(-1.95,0.34)	(-1.95,-0.34)	(-4.09,0)	(-5011.5,0)	(-5016.8,0)
∞	(-2,0)	(-4,0)	(-2,0)	(02,0)	(-4,0)	(- ∞,0)	(- ∞,0)

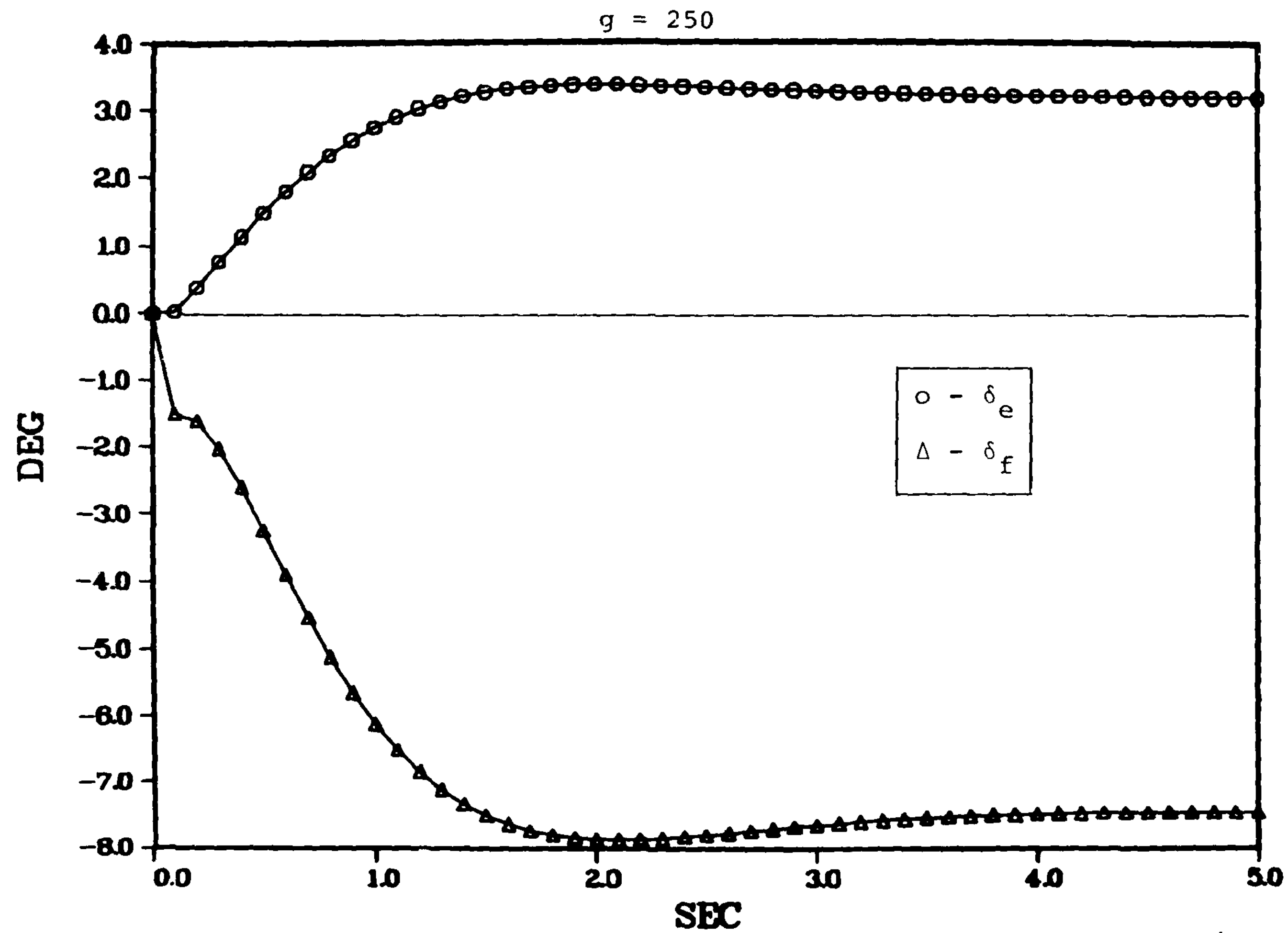

Fig. 4-36. Control Histories for Case B-3, Pitch Pointing Maneuver with Multipliers {1,0,0,1,1,0} (g = 250)

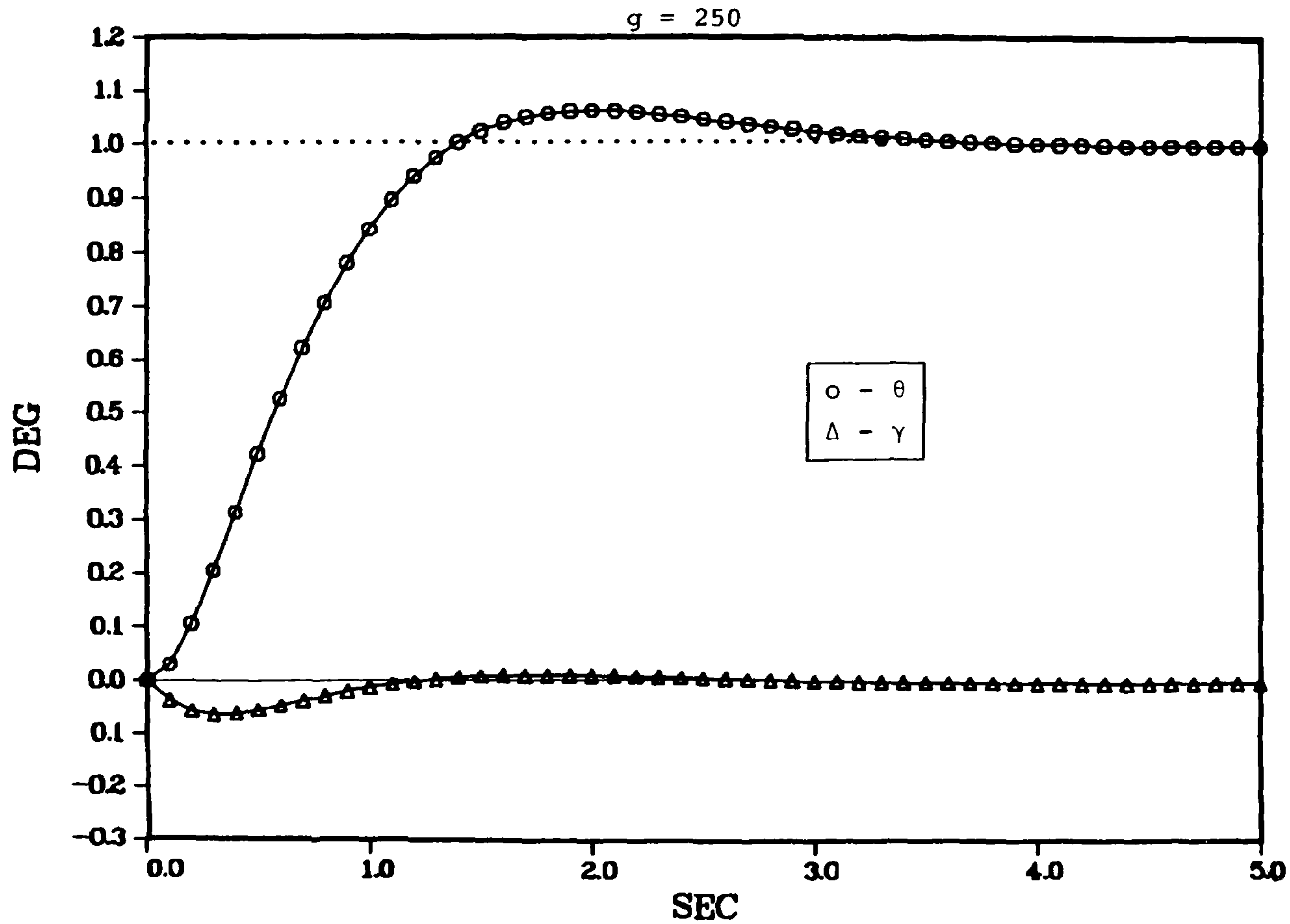

Fig. 4-37. Time Responses for Case B-3, Pitch Pointing Maneuver with Multipliers {1,0,0,1,1,0}

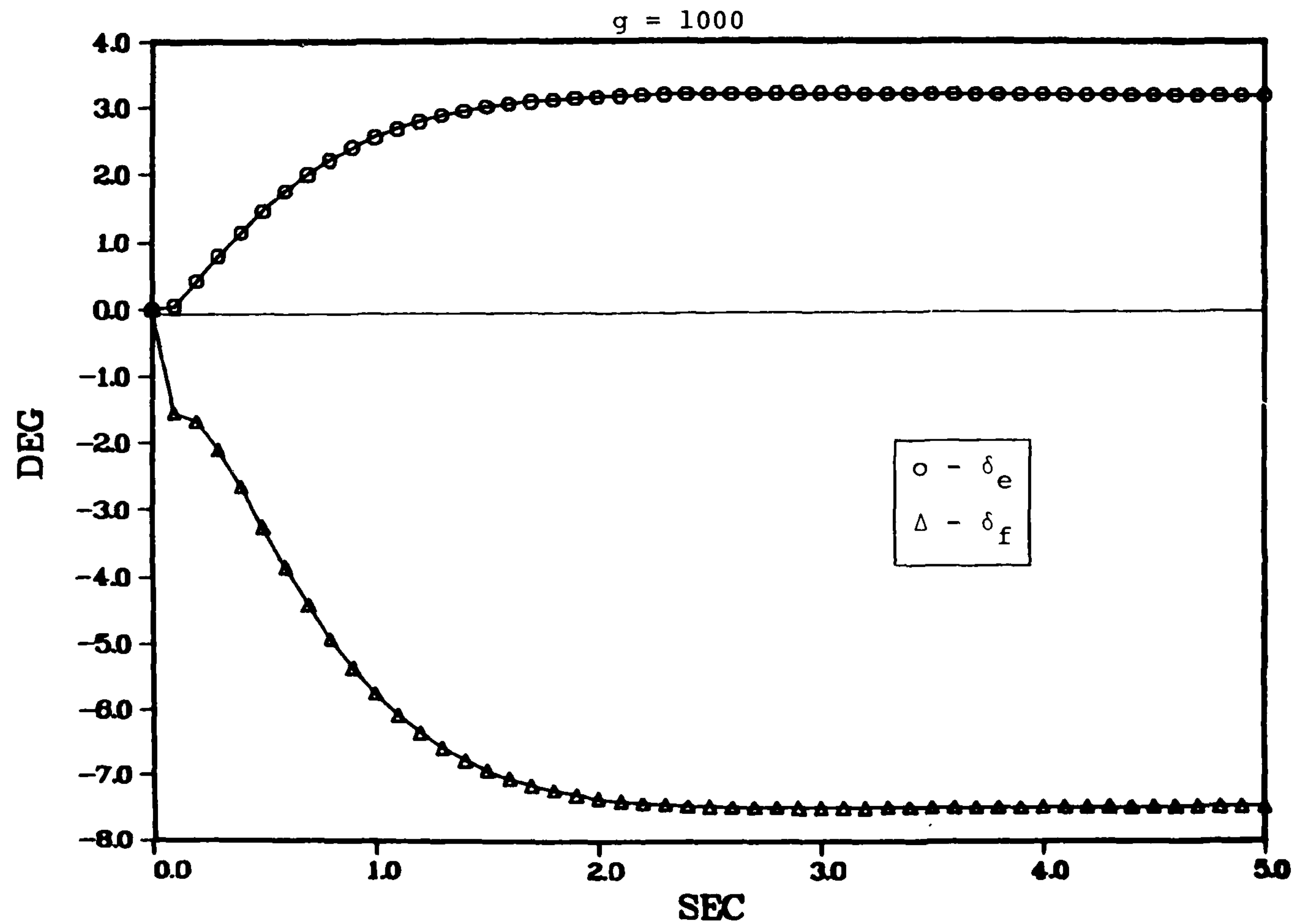

Fig. 4-38. Control Histories for Case B-3, Pitch Pointing Maneuver with Multipliers {1,0,0,1,1,0} (g = 1000)

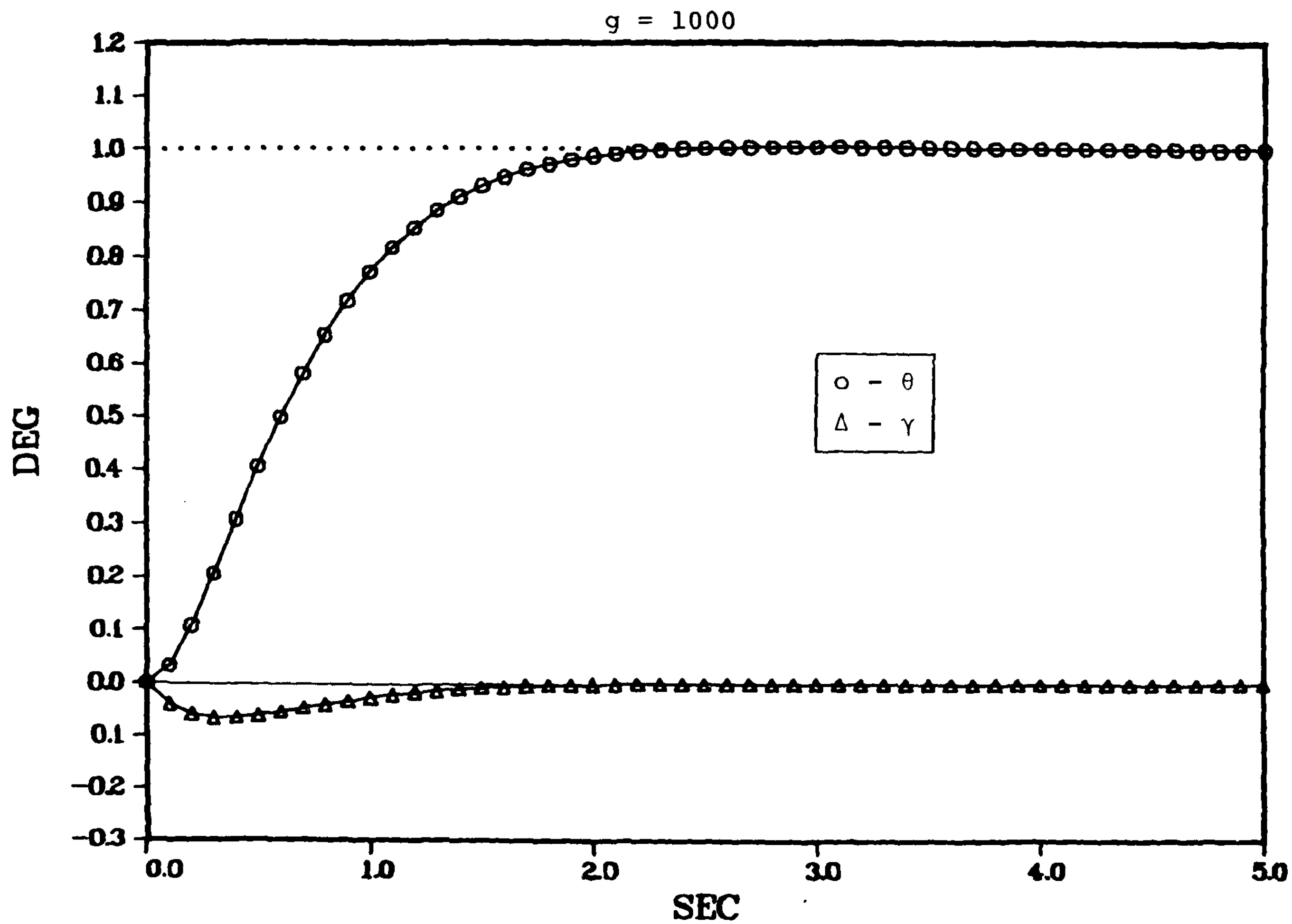

Fig. 4-39. Time Responses for Case B-3, Pitch Pointing Maneuver with Multipliers {1,0,0,1,1,0} (g = 1000)

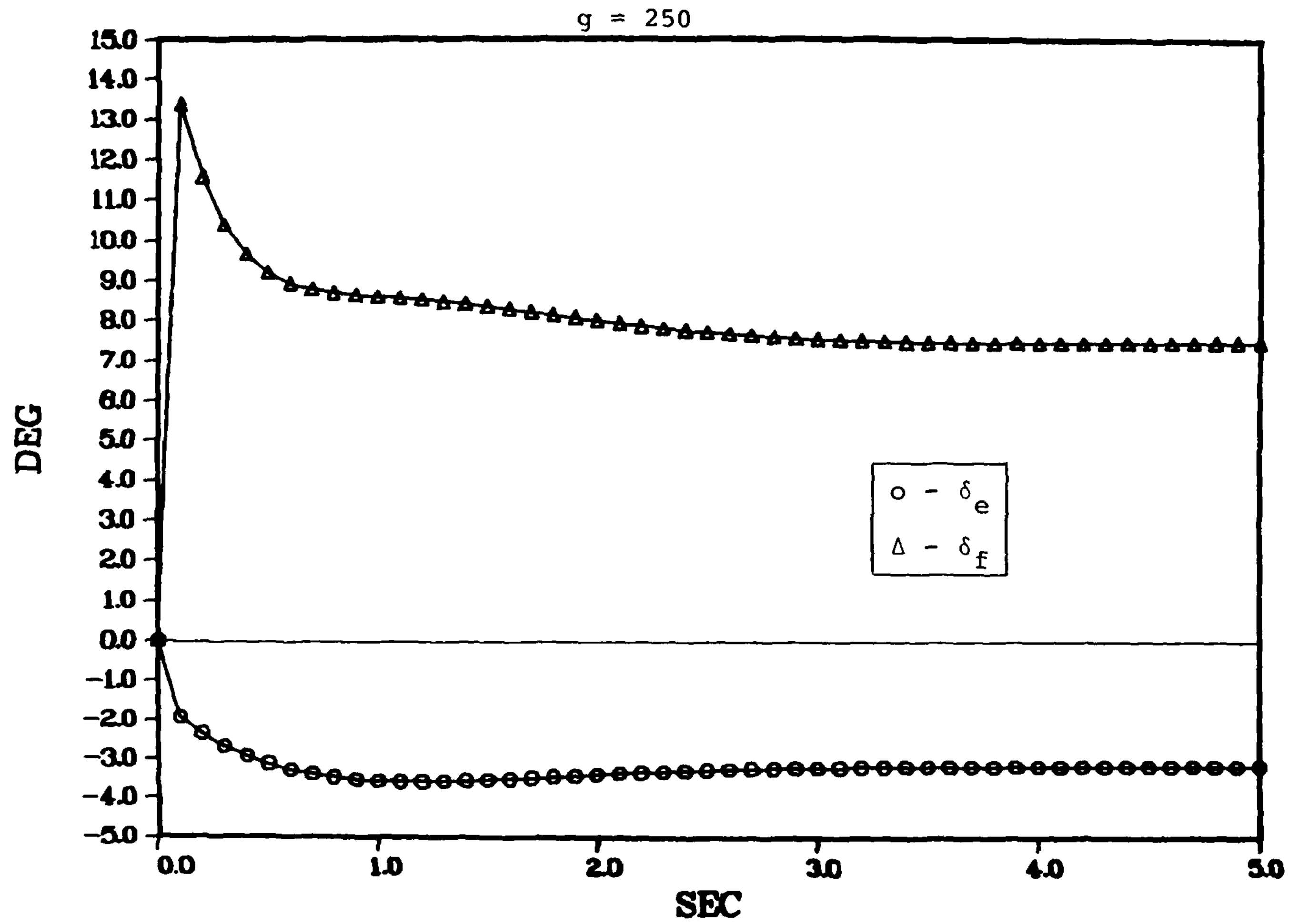

Fig. 4-40. Control Histories for Case B-3, Vertical Translation Maneuver with Multipliers {1,0,0,1,1,0} (g = 250)

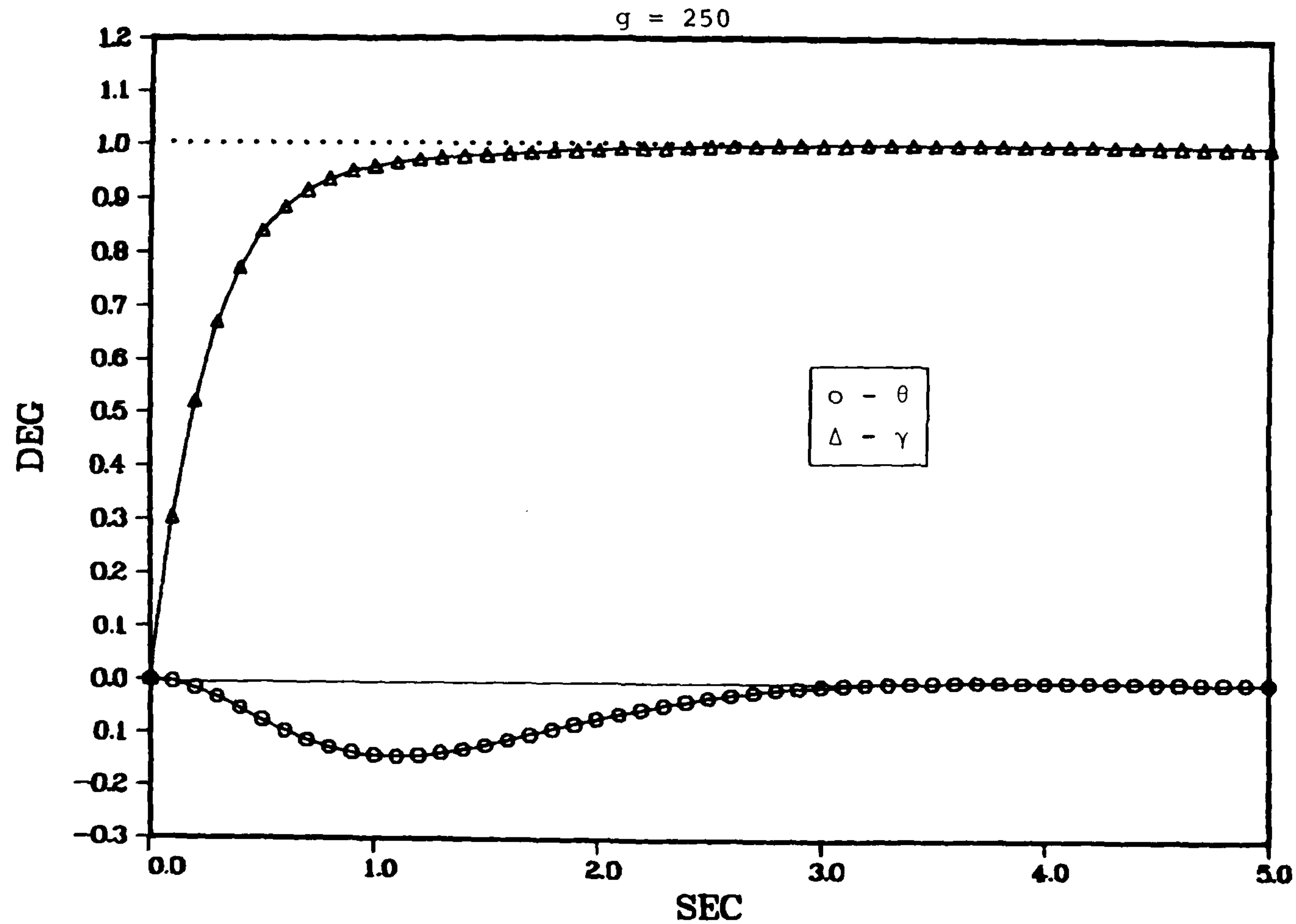

Fig. 4-41. Time Responses for Case B-3, Vertical Translation Maneuver with Multipliers {1,0,0,1,1,0} (g = 250)

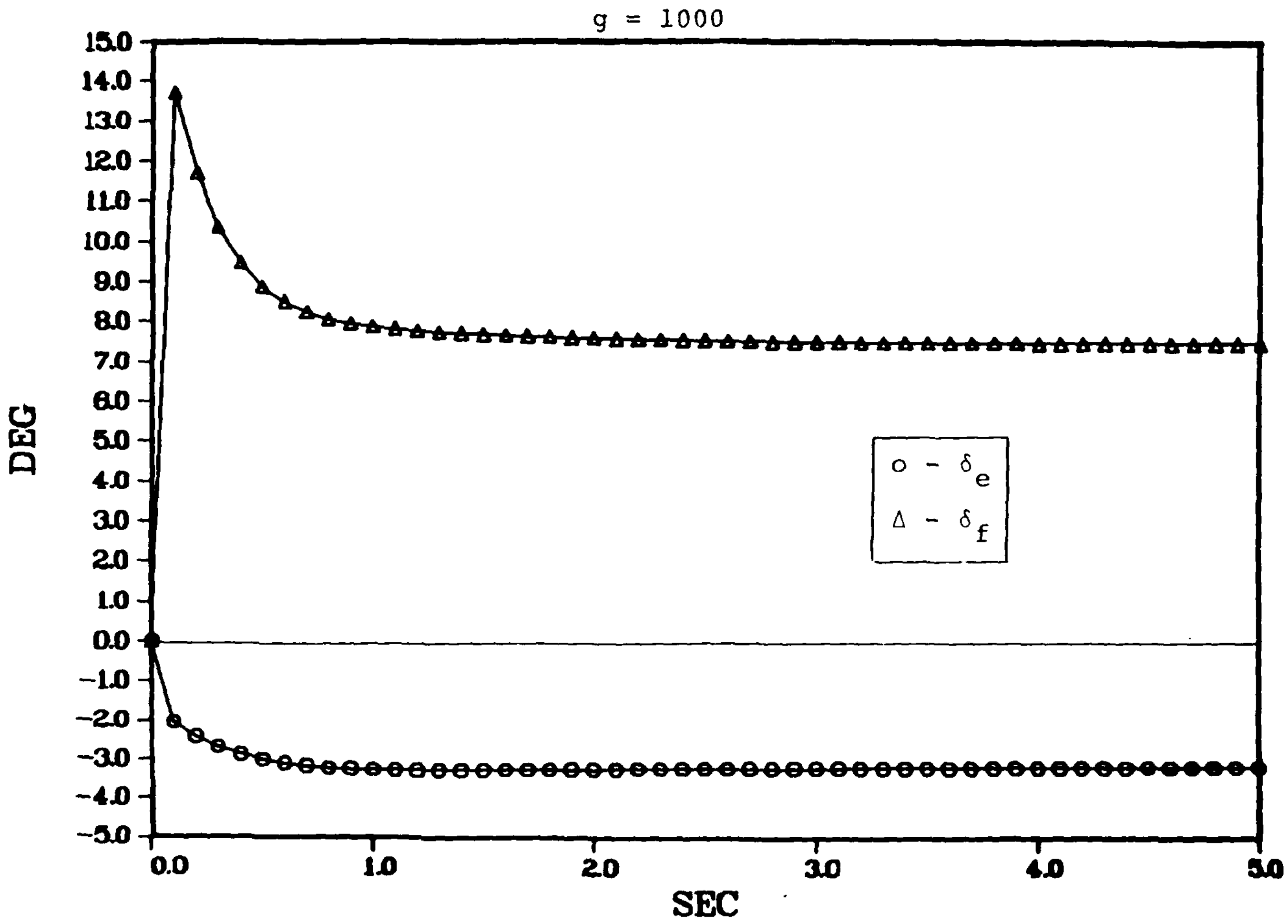

Fig. 4-42. Control Histories for Case B-3, Vertical Translation Maneuver with Multipliers {1,0,0,1,1,0} (g = 1000)

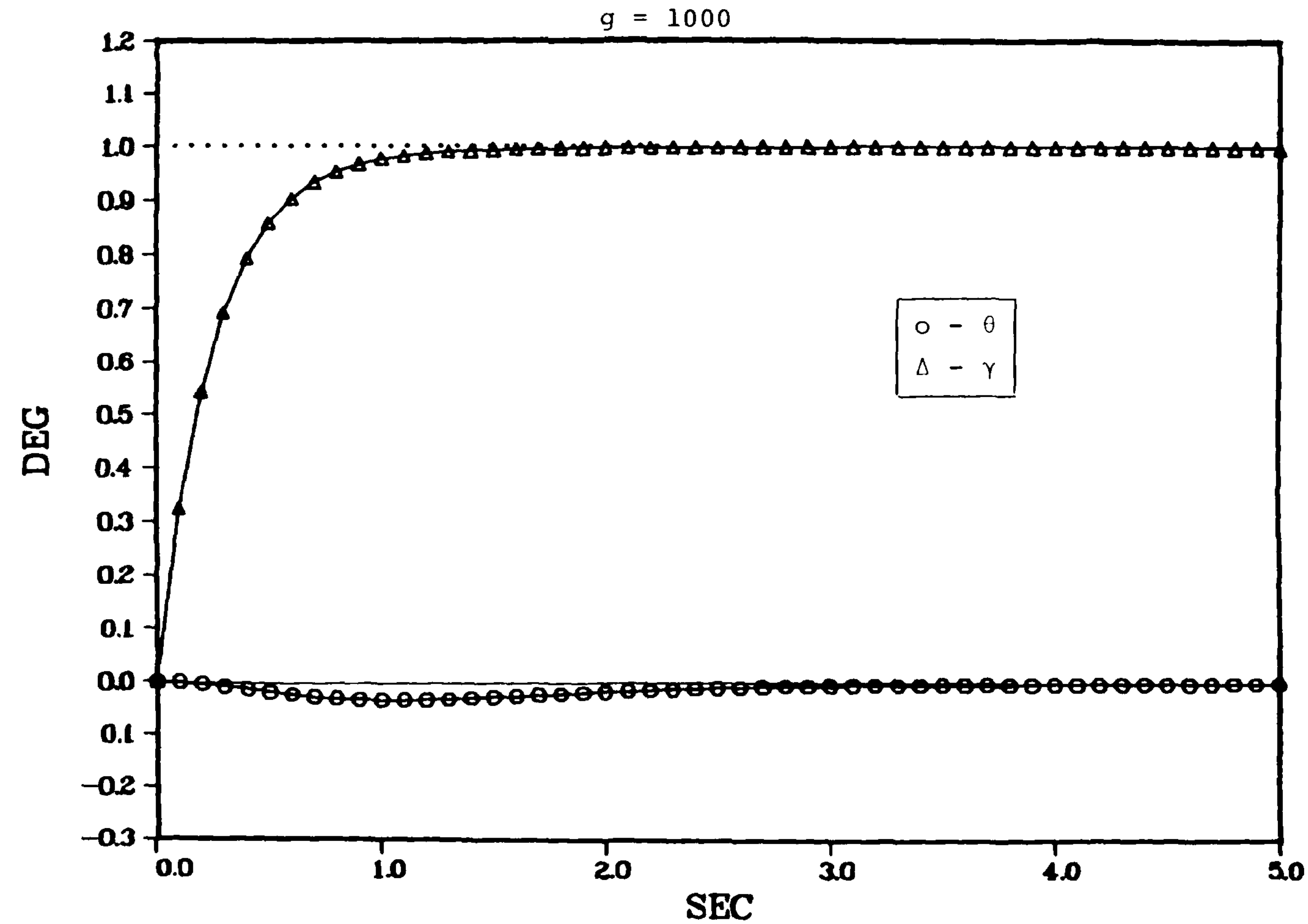

Fig. 4-43. Time Responses for Case B-3, Vertical Translation Maneuver with Multipliers {1,0,0,1,1,0} (g = 1000)

Case B-4

This case is identical to the last (same eigenvalues and eigenvectors) except that the output equations of equation (4-16) are used to achieve the direct lift control mode. The system matrices are shown in Fig. 4-44, and the eigenvalue movements are the same as in Table 4-10. The control and output histories for $g = 1000$ are shown in Figs. 4-45 and 4-46, respectively. The responses clearly show a significant amount of interaction in the first two seconds. To improve this, the eigenvectors would have to be modified, but this was not done here.

System C - Full Aircraft Model With Control Actuator Dynamics

In this group of tests the full longitudinal aircraft model is used (that is, the u state is added). The state equations of Fig. 3-5 were implemented originally, but since α is desired in the output equations, these equations are transformed to replace w with α by the relationship of equation (3-2). This transformation results in the following partitioned matrices.

$$A_{11} = \begin{bmatrix} -0.01505 & -12.823 & 0 & -32.174 \\ -0.0001 & -1.3411 & -0.99335 & 0 \\ 0.00018 & 43.233 & -0.86939 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$A_{12} = \begin{bmatrix} -2.5157 & -13.136 \\ -0.16897 & -0.25183 \\ -17.251 & -1.5766 \\ 0 & 0 \end{bmatrix} \qquad A_{12} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad (4\text{-}17)$$

$$M = \begin{bmatrix} 0.3181 & 0.017 & -0.25 \\ 0.4319 & 0.108 & 0.25 \end{bmatrix}$$

$$[F_1 \mid F_2] = \left[\begin{array}{ccc|cc} 1 & 0.0549 & 0.0708 & -0.2513 & 0.0361 \\ 0 & 0.5864 & 5.3333 & -1.9051 & -0.2332 \end{array}\right]$$

$$F_2B_2 = \begin{bmatrix} -5.025 & 0.7227 \\ -38.1025 & -4.6642 \end{bmatrix}$$

$$\Sigma = \text{diag}\ \{1,1\}$$

$$K_o = \begin{bmatrix} -0.0915 & -0.0142 \\ 0.7475 & -0.0986 \end{bmatrix} = \frac{1}{2}\ K_1$$

Fig. 4-44. System Matrices for Case B-4, Multipliers {1,0,0,1,1,0}

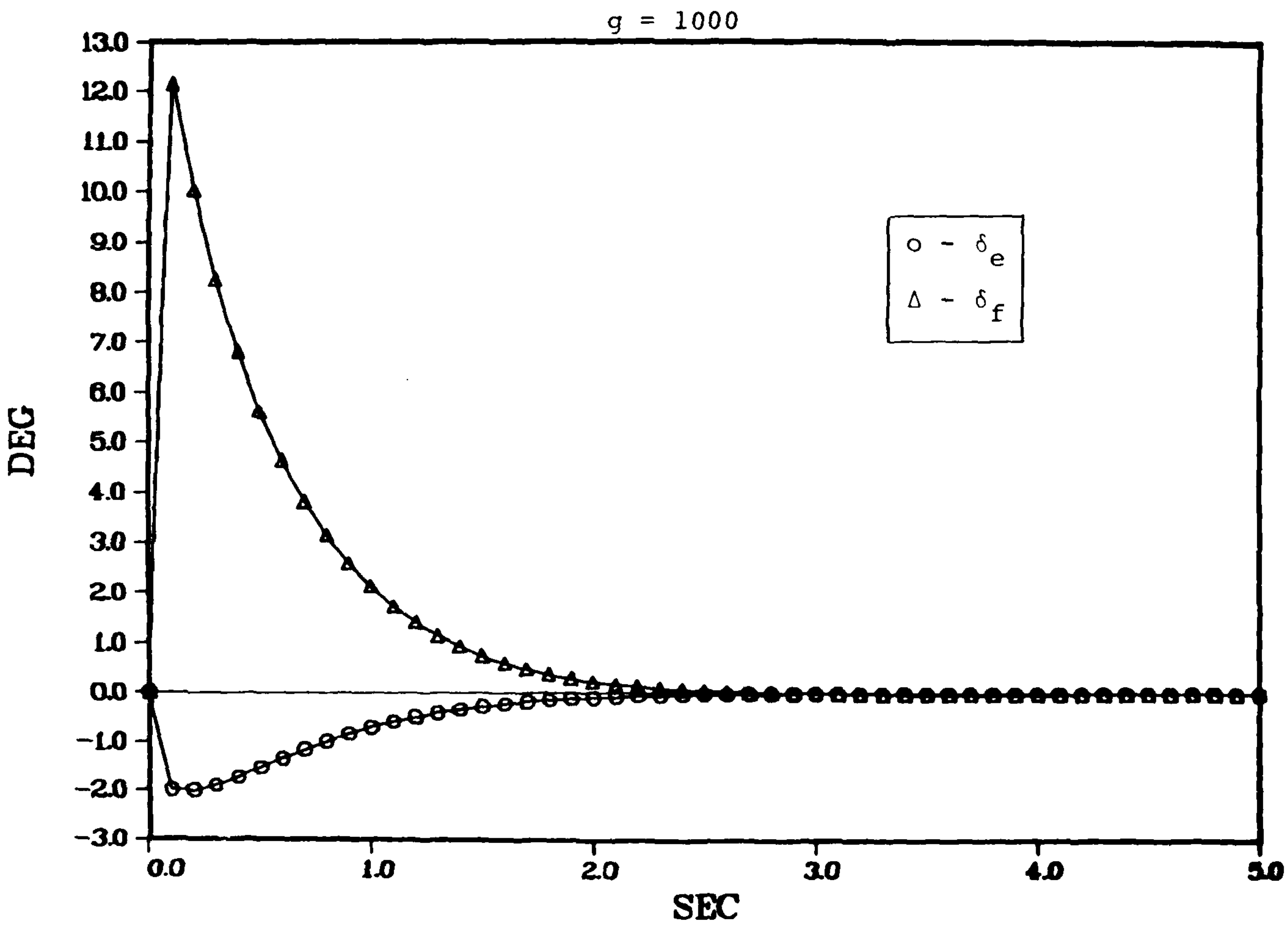

Fig. 4-45. Control Histories for Case B-4, Direct Lift Control Maneuver with Multipliers {1,0,0,1,1,0}

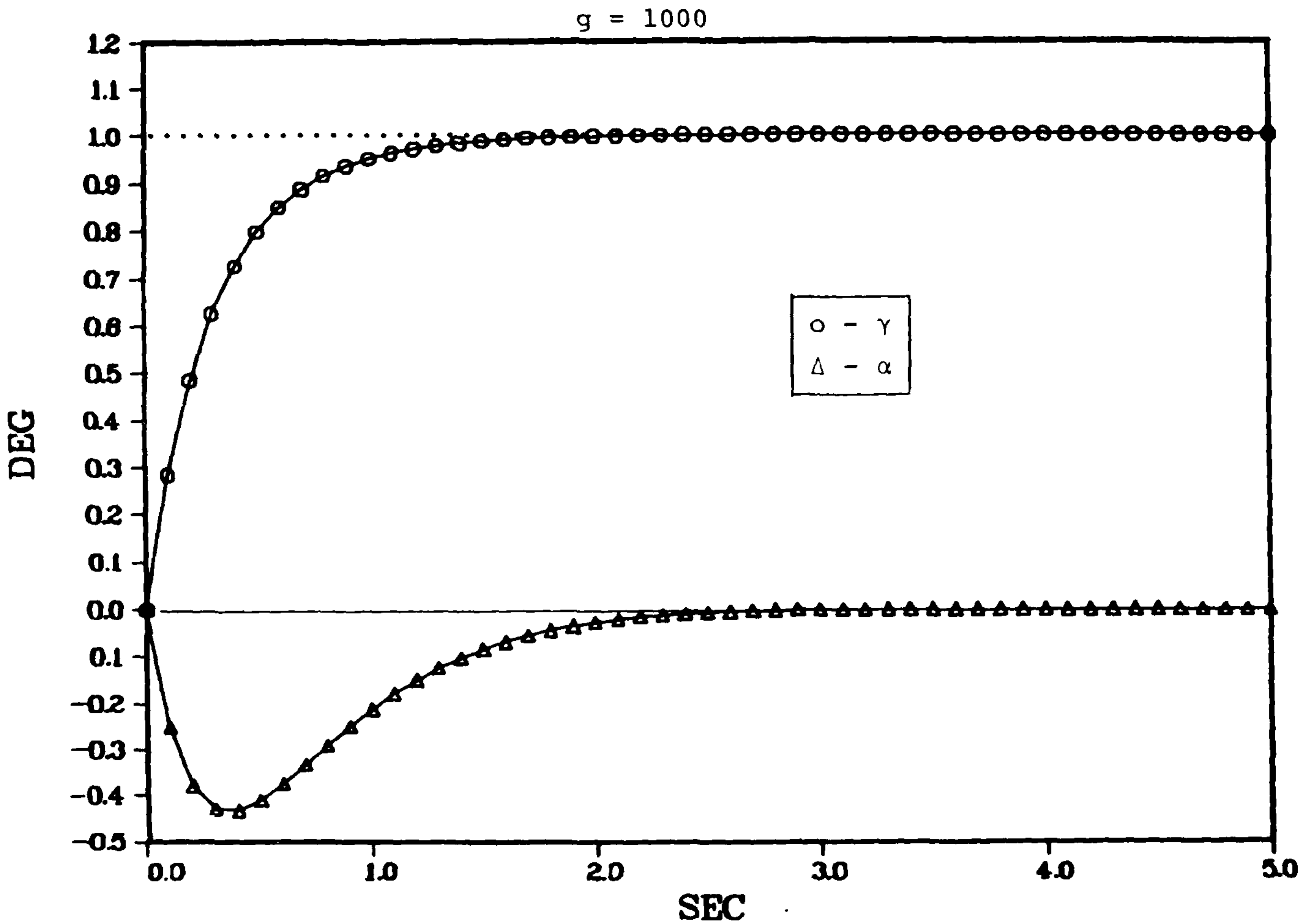

Fig. 4-46. Time Responses for Case B-4, Direct Lift Control Maneuver with Multipliers {1,0,0,1,1,0}

$$A_{22} = \begin{bmatrix} -20 & 0 \\ 0 & -20 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad B_2 = \begin{bmatrix} 20 & 0 \\ 0 & 20 \end{bmatrix} \tag{4-18}$$

It is important to note here that if the control actuator dynamics had not been modeled, the plant would not have been in the proper form for implementation (see equation (2-1)) and a transformation would have been necessary. Therefore, modeling control actuator dynamics guarantees that the plant is in the proper form for use of this method.

The output equations required for pitch pointing and vertical translation maneuvers are given by

$$[C_1 \,\vdots\, C_2]\underline{x}(t) = [\theta \;\; \gamma]^T = \left[\begin{array}{cccc|cc} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \end{array}\right] \underline{x}(t) \tag{4-19}$$

and those for direct lift control by

$$[C_1 \,\vdots\, C_2] = [\gamma \;\; \alpha]^T = \left[\begin{array}{cccc|cc} 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{array}\right] \tag{4-20}$$

The matrix C_2 is obviously rank deficient in both cases, thus again requiring extra plant measurements.

For these tests, four slow eigenvalues had to be selected, thus requiring eight constant multipliers to define the closed-loop eigenvectors.

Case C-1

Numerous test cases were completed using this system. Since all the results are similar, only one test case is developed. The four chosen eigenvalues are {(-4±3j); (-4+j0); (-3+j0)}, with output matrices given by equation (4-19). The first selection of null space vector multipliers is {1,0,0,0,1,0,1,0}. The system matrices are shown in Fig. 4-47, and the movements in Table 4-11. Note that the determinant of the F_2B_2 matrix is very small, therefore causing the K_o matrix to have extremely large terms (relation given in equation (2-38)). The response for this system is totally unrealizable. Due to the magnitude of the response no plots are shown, but the results for g = 1000 require thousands of degrees of control deflections and have output response overshoots as high as 700 degrees for $\underline{v} = [1\ 0]^T$deg and 1200 degrees for $\underline{v} = [0\ 1]^T$deg. After roughly six seconds the control deflections do settle out to reasonable values and the outputs assume their commanded values.

For multipliers {1,10,0,0,0,1,0,1} the system matrices are shown in Fig. 4-48 and eigenvalue movements in Table 4-12. The control deflections and output responses for this choice are still unrealizable but are reduced in peak magnitude by roughly a factor of ten. No combination of multipliers was found which produced a reasonable response, but the amount of testing done was far from exhaustive.

$$M = \begin{bmatrix} 0.069 & -1.3669 & -0.2331 & -0.3511 \\ 0.0531 & -1.5032 & -0.1305 & 0.5043 \end{bmatrix}$$

$$F_1 = \begin{bmatrix} -0.0009 & -9.1302 & -1.5063 & -1.22 \\ -0.0007 & -5.3069 & -0.8755 & -0.7084 \end{bmatrix}$$

$$F_2 = \begin{bmatrix} 4.079 & -0.1946 \\ 2.3717 & -0.1132 \end{bmatrix}$$

$$F_2B_2 = \begin{bmatrix} 81.5799 & -3.892 \\ 47.4346 & -2.2634 \end{bmatrix} \quad \det F_2B_2 = -0.038$$

$$\Sigma = \mathrm{diag}\ \{1,1\}$$

$$K_o = \begin{bmatrix} 59.3948 & -102.1286 \\ 1244.7247 & -2140.7293 \end{bmatrix} = \frac{1}{2} K_1$$

Fig. 4-47. System Matrices for Case C-1, Multipliers {1,0,0,0,1,0,1,0}

Table 4-11. Eigenvalue Movements for Case C-1, Multipliers {1,0,0,0,1,0,1,0}

gain,g	Eigenvalues							
0	(0,0)	(0,0)	(-0.0076,0.068)	(-0.0076,-0.068)	(5.45,0)	(-7.66,0)	(-20,0)	(-20,0)
0.22	(-0.004,0)	(3.33,0.46)	(-0.61,0.29)	(-0.61,-0.29)	(3.33,-0.46)	(-7.66,0)	(-20.06,0)	(20.38,0)
0.65	(-0.01,0)	(3.53,3.01)	(-0.82,0.06)	(-0.82,-0.06)	(3.53,-3.01)	(-7.66,0)	(-20.17,0)	(-21.11,0)
1	(-0.02,0)	(3.60,3.90)	(-0.67,0)	(-1.13,0)	(3.60,-3.90)	(-7.66,0)	(-20.27,0)	(-21.68,0)
10	(-0.19,0)	(2.53,10.33)	(-0.51,0)	(-1.83,0)	(2.53,-10.33)	(-7.61,0)	(-23.12,0)	(-34.02,0)
15	(-0.37,0.06)	(1.54,11.43)	(-0.37,-0.06)	(-1.96,0)	(1.54,-11.43)	(-7.58,0)	(-25.09,0)	(-40.01,0)
50	(-0.49,0.41)	(-2.95,11.18)	(-0.49,-0.41)	(-1.96,0)	(-2.95,-11.18)	(-7.40,0)	(-48.00,0)	(-77.98,0)
100	(-0.63,0.56)	(-4.12,8.95)	(-0.63,-0.56)	(-1.98,0)	(-4.12,-8.95)	(-7.21,0)	(-94.22,0)	(-129.33,0)
500	(-1.17,0.80)	(-4.23,5.42)	(-4.23,-5.42)	(-2.00,0)	(-1.17,-0.80)	(-6.46,0)	(-492.1,0)	(-530.9,0)
1000	(-1.17,0.81)	(-4.15,4.58)	(-4.15,-4.58)	(-2.00,0)	(-1.37,-0.81)	(-6.06.0)	(-991.9,0)	(-1031.2,0)
5000	(-1.91,0.65)	(-4.04,3.51)	(-4.04,-3.51)	(-2.00,0)	(-1.91,-0.65)	(-5.12.0)	(-9991,0)	(-10031,0)
∞	(-3,0)	(-4,3)	(-4,-3)	(-2,0)	(-2,0)	(-4,0)	(- ∞,0)	(- ∞,0)

$$M = \begin{bmatrix} -0.002 & 0.4419 & -0.0071 & -0.3414 \\ 0.0117 & -0.4499 & 0.001 & 0.5087 \end{bmatrix}$$

$$F_1 = \begin{bmatrix} -0.0000 & -0.8735 & 0.1037 & 1.0652 \\ -0.0001 & -0.5041 & 0.0609 & 0.6223 \end{bmatrix}$$

$$F_2 = \begin{bmatrix} 0.0529 & -0.0735 \\ 0.0293 & -0.0425 \end{bmatrix}$$

$$F_2B_2 = \begin{bmatrix} 1.0581 & -1.4696 \\ 0.5857 & -0.8495 \end{bmatrix} \quad \det F_2B_2 = -0.038$$

$$\Sigma = \mathrm{diag}\ \{1,1\}$$

$$K_o = \begin{bmatrix} 22.2911 & -38.5627 \\ 15.3687 & -27.7643 \end{bmatrix} = \frac{1}{2} K_1$$

Fig. 4-48. System Matrices for Case C-1, Multipliers {1,10,0,0,0,1,0,1}

Table 4-12. Eigenvalue Movements for Case C-1, Multipliers {1,10,0,0,0,1,0,1}

gain,g	Eigenvalues							
0	(0,0)	(0,0)	(-0.0076,0.068)	(-0.0076,-0.068)	(5.45,0)	(-7.66,0)	(-20,0)	(-20,0)
0.2	(-0.27,0)	(0.67,0)	(-0.12,0.03)	(-0.12,-0.03)	(5.16,0)	(-7.67,0)	(-20.10,0)	(-20.20,0)
1.1	(-0.75,0)	(2.93,0.28)	(-0.13,0.22)	(-0.13,-0.22)	(2.93,-0.28)	(-7.68,0)	(-20.52,0)	(-21.06,0)
5	(-1.27,0)	(2.34,3.83)	(-0.24,0.46)	(-0.24,-0.46)	(2.34,-3.83)	(-7.70,0)	(-24.85,0)	(-22,60,0)
25	(-1.74,0)	(-0.39,6.09)	(-0.59,0.77)	(-0.59,-0.77)	(-0.39,-6.09)	(-7.52,0)	(-36.34,0)	(-44.65,0)
100	(-1.92,0)	(-2.85,5.24)	(-1.10,0.90)	(-1.10,-0.90)	(-2.85,-5.24)	(-6.89,0)	(-106.00,0)	(-119.52,0)
500	(-1.98,0)	(-3.69,3.88)	(-1.69,0.79)	(-1.69,-0.79)	(-3.69,-3.88)	(-5.81,0)	(-504.22,0)	(-519.46,0)
1000	(-1.99,0)	(-3.82,3.53)	(-1.90,0.68)	(-1.90,-0.68)	(-3.82,-3.53)	(-5.35,0)	(-1004.0,0)	(-1019.5,0)
10,000	(-2.00,0)	(-3.97,3.07)	(-2.56,0)	(-215,0)	(-3.97,-3.07)	(-4.33,0)	(-10003,0)	(-10019,0)
∞	(-2,0)	(-4,3)	(-3,0)	(-2,0)	(-4,-3)	(-4,0)	(- ∞,0)	(- ∞,0)

System D - Robustness Tests

In order to examine how well this method behaves when variations in the system plant are encountered, a series of robustness tests are performed. Appendix A-2 shows the values of the aircraft stability derivatives when the design altitude and Mach number are changed from h = 3000 ft, M = 0.60 to h = 10,000 ft, M = 0.70. Also, in the test cases encorporating control actuator dynamics, the actuator transfer functions are also changed from that of equation (3-3) to

$$\frac{\delta}{\delta_c} = \frac{15}{s+15} \tag{4-21}$$

To demonstrate robustness, these altered plants are used with the controller and transducer matrices found in all of the previous test cases, except for the full model test cases where no satisfactory results were obtained.

The aircraft plants used in these tests are found in Appendix A-2. In all of the tests for robustness, no noticeable change in time histories occurs. No plots are shown because they appear to be exact duplicates of the design condition results.

V. Conclusions and Recommendations

Nine major conclusions are drawn from the results presented in this study. A number of these conclusions suggest that a great deal more study is necessary before all the effects of the various aspects of this method are fully understood. The conclusions are presented in a designer's order, that is, they try to answer the questions that would arise as the controller design process proceeds.

The major effect of the selection of the slow eigenvalues, beside the fact that they insure stability at high gain when chosen to be in the left-half plane, is that their selection changes the magnitude of the F_2B_2 matrix. For any given set of selected slow eigenvalues, the determinant of F_2B_2 does not change for different eigenvector choices. Since K_o is found by inverting F_2B_2, the larger the value of the determinant, the smaller the terms in K_o and K_1 will be. The size of the terms in K_o has a major effect on the response, as seen in many of the test cases.

The selection of the slow eigenvectors directly affects the transient response of the outputs. The effect of the choice of the eigenvectors is best seen in Cases A-1, A-3, and B-1. Different eigenvectors produce different M matrices and therefore different measurement matrices, F_1 and F_2. Although the determinant of F_2B_2 remains constant for any choice of eigenvectors, the

elements of F_2B_2 change and thus the elements of K_o and K_1 change. These changes in the controller matrices radically alter the transient response. It has been observed in all the test cases which showed different eigenvector choices that a system with larger values of K_o and K_1 has a poorer response than one with smaller values in these matrices.

The choice of the elements in the Σ matrix was briefly examined in Case A-6. The only effect of changing the values of Σ was to raise or lower the value of g by the inverse of the amount by which σ was changed. That is, if Σ is decreased by a factor of ten, the responses are identical except that the value of g must be increased by a factor of ten. None of the test cases examined the effect of choosing $\sigma_1 \neq \sigma_2$ or changing one value of σ while holding the other constant.

Changing the ratio between K_o and K_1 has a small but noticeable effect on the response. The ratio of K_1 to K_o defines the first set of slow eigenvalues (see equation (2-39)). Test cases A-1 and A-3 along with A-2 and A-4 are identical except that in A-1 and A-2, $K_o = K_1$ and in A-3 and A-4, $K_1 = 2K_o$. The effect of this change is to slightly decrease the final value overshoots and settling times, which suggests the first set of slow eigenvalues are slightly visible in the response.

The gain parameter, g, has the single most pronounced effect on the controller performance. In all the aircraft plants used in this study, the open-loop

response is unstable due to a right-half plane pole. As g increases, one of the integrator poles (which is at the origin in the open-loop) moves into the right-half plane, meets with the unstable pole, and they then split off the real axis and move back toward the left-half plane. At some value of g they enter the left-half plane and the system becomes stable. As g is increased further, the system poles approach their asymptotic values. However, as g is made very large, the terms in the fast state differential equations become very large and integrating them becomes both costly and inaccurate. In several of the cases in this report, very large gains were tried, which resulted in the computed response degrading from the results shown. This is possibly the result of trying to numerically integrate a system of equations that includes "stiff" modes.

In all of the B test cases and in the full model tests, control actuator dynamics are modeled in the aircraft state equations. The responses in the A test cases are too fast--modeling actuator dynamics in the B cases slowed the responses to an acceptable level. In the full model cases the basic plant (without actuator dynamics) is not in the form required to apply Porter's method. By introducing actuator dynamics, one can avoid the necessity of transforming the state equations since the plant is automatically put into the proper form.

The robustness tests demonstrate the most attractive feature of this method. In all cases, when the flight condition is slightly changed and the model of the control actuators altered, the original controller and transducer matrices still produced the desired flight modes. Not only are these modes satisfactorily effected, the response is practically identical to that of the original system.

Unfortunately, not all desired outputs can be tracked with this method. For example, when pitch rate q, or normal acceleration A_N, is a desired output, it is not possible to find an M matrix which satisfies the necessary conditions (that the second set of slow eigenvalues are all in the left-half plane and that the matrix F_2 has full rank).

The most accurate outputs to describe the three modes in this study are not θ, γ, and α, but q, A_N, and α. Since q and A_N outputs yield uncontrollable systems, they were replaced with θ and γ as approximations. The reason for this failure is that the system matrix for these outputs is not fully controllable, so the method will not work. These cases (when q or A_N was used) represented the only ones in which F_2 was not full rank.

The final conclusion deals with the full model of the aircraft. For some reason, it is extremely difficult in this case to choose the eigenvalues, the eigenvectors, the Σ matrix, and a ratio of K_o and K_1 that produces a satisfactory response. The only major trend noticeable

here is that the response is drastically worse when the controller matrices contain very large terms.

Recommendations

The full effects of all the design parameters in this method still require a great deal of investigation. Particularly, the effect of the chosen eigenvalues and eigenvectors on the response still requires more study since the results clearly indicate they do appear in the closed-loop system. Also, the effects of different choices of Σ should be examined further. The problems with the full model cases require particular attention. It would be highly beneficial to the advancement of this method if a standard way to choose the various design parameters to minimize the amount of commanded value overshoot and interaction could be devised. Since choosing different eigenvectors greatly affects the transient responses, it is possible that the work being done on entire eigenstructure assignment problems may be applicable to this method.

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Appendix A-1. Data for the AFTI-16 Aircraft

Physical Description of the Aircraft

Mass = 653 slugs $\qquad \bar{C} = 11.32$ ft

$S = 300\ ft^2 \qquad I_y = 53876\ slug/ft^2$

$b = 30$ ft

Flight Condition Data

$M = 0.60 \qquad a = 1104\ ft/sec^2$

$h = 3{,}000$ ft $\qquad U = 663$ ft/sec

$\rho = 0.002177\ slug/ft^3 \qquad \bar{q} = 478\ lb/ft^2$

Non-Dimensional Stability Derivatives

$\alpha_o = 0.0336$ rad $\qquad C_{D_o} = 0.0227$

$C_{L_o} = 0.1459 \qquad C_{M_o} = 0.1617$

$C_{L_\alpha} = 4.0148\ rad^{-1} \qquad C_{M_\alpha} = 0.1443\ rad^{-1}$ (-3.6% static margin)

$C_{D_\alpha} = 0.2043\ rad^{-1}$

$C_{L_{\dot\alpha}} = -0.9632\ rad^{-1} \qquad C_{M_{\dot\alpha}} = -0.7853\ rad^{-1}$

$C_{L_{\delta_e}} = 0.5087\ rad^{-1} \qquad C_{M_{\delta_e}} = -0.5735\ rad^{-1}$

$C_{D_{\delta_e}} = 0.0115\ rad^{-1}$

$C_{L_{\delta_f}} = 0.7582\ rad^{-1} \qquad C_{M_{\delta_f}} = -0.0540\ rad^{-1}$

$C_{D_{\delta_f}} = 0.0598\ rad^{-1}$

Stability Axes-Dimensional Derivatives

$$X_u = -\frac{\rho S U}{M} C_{D_o} = -0.0151\ sec^{-1}$$

$$Z_u = -\frac{\rho SU}{M} C_{L_o} = -0.0967 \text{ sec}^{-1}$$

$$M_u = \frac{\rho SU\bar{C}}{Iy} C_{M_o} = 1.47 \times 10^{-4} \text{ ft}^{-1}\text{-sec}$$

$$X_w = \frac{\rho SU}{2M} (C_{L_o} - C_{D_\alpha}) = -0.0194 \text{ sec}^{-1}$$

$$Z_w = -\frac{\rho SU}{2M} (C_{L_\alpha} + C_{D_o}) = -1.3375 \text{ sec}^{-1}$$

$$M_w = \frac{\rho SU\bar{C}}{2Iy} C_{M_\alpha} = 0.06564 \text{ ft}^{-1}\text{-sec}^{-1}$$

$$X_{\dot{w}} = 0$$

$$Z_{\dot{w}} = -\frac{\rho S\bar{C}}{4M} C_{L_{\dot{\alpha}}} = 2.73 \times 10^{-3} \text{ rad}^{-1}$$

$$M_{\dot{w}} = \frac{\rho S\bar{C}^2}{4Iy} C_{M_{\dot{\alpha}}} = -3 \times 10^{-4} \text{ft}^{-1}$$

$$X_q = 0$$

$$Z_q = -\frac{\rho SU\bar{C}}{4M} C_{L_q} = -4.4098 \text{ ft-sec}^{-1}$$

$$M_q = \frac{\rho SU\bar{C}^2}{4Iy} C_{M_q} = -0.6719 \text{ sec}^{-1}$$

$$X_{\delta_e} = -\frac{\rho U^2 S}{2M} C_{D_{\delta_e}} = -2.5157 \text{ ft-sec}^{-2}$$

$$Z_{\delta_e} = -\frac{\rho U^2 S}{2M} C_{L_{\delta_e}} = -111.66 \text{ ft-sec}^{-2}$$

$$M_{\delta_e} = \frac{\rho U^2 S\bar{C}}{2Iy} C_{M_{\delta_e}} = -17.2845 \text{ sec}^{-2}$$

$$X_{\delta_f} = -\frac{\rho U^2 S}{2M} C_{D_{\delta_f}} = -13.132 \text{ ft-sec}^{-2}$$

$$Z_{\delta_f} = -\frac{\rho U^2 S}{2M} C_{L_{\delta_f}} = -166.426 \text{ ft-sec}^{-2}$$

$$M_{\delta_f} = \frac{\rho U^2 S\bar{C}}{2Iy} C_{M_{\delta_f}} = -1.6267 \text{ sec}^{-2}$$

Appendix A-2. Robustness Test Models

Physical Description of the Aircraft

Mass = 653 slugs

$\bar{C}$ = 11.32 ft

S = 300 ft^2

Iy = 53876 slug/ft^2

b = 30 ft

Flight Condition Data

M = 0.70

a = 1077 $ft\text{-}sec^{-2}$

h = 10,000 ft

U = 754 $ft\text{-}sec^{-2}$

ρ = 0.001756 $slug\text{-}ft^{-3}$

$\bar{q}$ = 499 $lb\text{-}ft^{-2}$

Non-Dimensional Stability Derivatives

Assumed to remain the same as in Appendix A-1 due to small flight condition perturbation.

Stability Axis-Dimensional Derivatives

$X_u = -0.01386 \ sec^{-1}$

$Z_u = -0.08875 \ sec^{-1}$

$M_u = 1.349 \times 10^{-4} \ ft^{-1}\text{-}sec^{-1}$

$X_w = -0.0178 \ sec^{-1}$

$Z_w = -1.2275 \ sec^{-1}$

$M_{\dot{w}} = 0.06024 \ ft^{-1} sec^{-1}$

$X_{\dot{w}} = 0$

$Z_{\dot{w}} = 2.20 \times 10^{-3} \ rad^{-1}$

$M_{\dot{w}} = -2.42 \times 10^{-4} \ ft^{-1}$

$X_q = 0$

$Z_q = -4.047 \text{ ft-sec}^{-1}$

$M_q = -0.6166 \text{ sec}^{-1}$

$X_{\delta_e} = -2.6268 \text{ ft-sec}^{-2}$

$Z_{\delta_e} = -116.592 \text{ ft-sec}^{-2}$

$M_{\delta_e} = -18.048 \text{ sec}^{-2}$

$X_{\delta_f} = -173.777 \text{ ft-sec}^{-2}$

$M_{\delta_f} = -1.699 \text{ sec}^{-2}$

Linear Model Without Control Actuators

$$\begin{bmatrix} \dot{\theta} \\ \dot{q} \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -0.7985 & 45.646 \\ 0 & 0.99683 & -1.2302 \end{bmatrix} \begin{bmatrix} \theta \\ q \\ \alpha \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -18.02 & -1.881 \\ -0.14833 & -0.23099 \end{bmatrix} \begin{bmatrix} \delta_e \\ \delta_f \end{bmatrix}$$

Linear Model With Control Actuators

$$\begin{bmatrix} \dot{\theta} \\ \dot{q} \\ \dot{\alpha} \\ \dot{\delta}_e \\ \dot{\delta}_f \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & -0.7985 & 45.646 & -18.02 & -1.881 \\ 0 & 0.99683 & -1.2302 & -0.14833 & -0.23099 \\ 0 & 0 & 0 & -15 & 0 \\ 0 & 0 & 0 & 0 & -15 \end{bmatrix} \begin{bmatrix} \theta \\ q \\ \alpha \\ \delta_e \\ \delta_f \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 15 & 0 \\ 0 & 15 \end{bmatrix} \begin{bmatrix} \delta_{e_C} \\ \delta_{f_C} \end{bmatrix}$$

Eigenvalues
(0,0)
(-7.76,0)
(5.74,0)
(-15,0)
(-15,0)

Appendix B. Discussion of the Usage of Null Space Vectors from a Singular Value Decomposition to Form Closed-Loop Eigenvectors

Real, Non-repeated Eigenvalues

When all the chosen slow eigenvalues are real, and none are repeated, the null space of $[\lambda_i I - A_{11},\ A_{12}]$ corresponding to each assigned eigenvalue contains ℓ vectors, each of which has length $n+\ell$ (ℓ is the number of inputs, n is the number of states). Therefore, each set of ℓ vectors must be linearly combined (requiring ℓ constant multipliers) to form each eigenvector. The only restriction placed upon the multipliers in this case is that the eigenvectors produced are linearly independent.

Complex Eigenvalues

The selection of complex eigenvalues results in complex eigenvectors. Since the singular value decomposition computer routine works only with real numbers, the complex eigenvectors must be selected from vectors lying in the null space of (Ref 6):

$$\begin{bmatrix} \lambda_{r_k} I - A_{11} & \lambda_{i_k} I & A_{12} & 0 \\ -\lambda_{i_k} I & \lambda_{r_k} I - A_{11} & 0 & A_{12} \end{bmatrix} \begin{bmatrix} \underline{\zeta}_{r_k} \\ \underline{\zeta}_{i_k} \\ \underline{\omega}_{r_k} \\ \underline{\omega}_{i_k} \end{bmatrix} = 0 \qquad \text{(B-1)}$$

where the subscripts r and i refer to real and imaginary, respectively. Using these vectors, the feedback gain matrix S is found from

$$S[\underline{\zeta}_{r_1},\underline{\zeta}_{i_1},\underline{\zeta}_3,\ldots,\underline{\zeta}_n] = [\underline{\omega}_{r_1},\underline{\omega}_{i_1},\underline{\omega}_3,\ldots\underline{\omega}_n] \qquad \text{(B-2)}$$

In this case, however, 2ℓ vectors lie in the null space of the matrix in (B-1) for each complex eigenvalue. This is due to the fact that complex eigenvalues come in pairs, with half of the vectors corresponding to the eigenvalue with the positive imaginary part and the other half to the eigenvalue with the negative imaginary part. Furthermore, each of these 2ℓ vectors has $2(n+\ell)$ elements, corresponding to the real and imaginary part of each. Therefore, the vectors should be combined with 2ℓ multipliers and then are separated to conform with equation (B-2). In this study, however, half of the null space vectors are given zero multipliers and the ℓ remaining vectors are combined and separated to form the closed-loop eigenvectors. This is found to correctly reproduce the chosen eigenvalues when the corresponding closed-loop A matrix is formed, so its use is justified.

Repeated Eigenvalues

If repeated eigenvalues are desired, two approaches are possible depending on how many repeated values are desired. If the number of desired repeated eigenvalues is less than or equal to the dimension of the null space (ℓ), then it is possible to choose linearly independent eigenvectors from these spaces. In this case, no extension to the previously developed theory is necessary, but the designer is more restricted in eigenvector

selection. One way to avoid choosing linearly dependent eigenvectors (the one used in this study) is to choose each eigenvector as a multiple of one of the null space vectors.

To assign more than ℓ repeated eigenvalues or to increase the flexibility in eigenvector selection for ℓ or less repetitions, a string of generalized eigenvectors must be generated. Since it is not used in this study, the reader is referred to Ref 6 for further details.

Appendix C. Main Computer Program and Example

```
      PROGRAM PORTER(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,
     1   CDATA,TAPE3=CDATA)
      REAL A11(8,8),A12(8,8),A21(8,8),A22(8,8)
      REAL B1(8,8),B2(8,8),C1(8,8),C2(8,8)
      REAL F1(8,8),F2(8,8),M(8,8),F2B2D(8,8)
      REAL K0(8,8),K1(8,8),X(8),Y(8),Z(8)
      REAL A12FF(8,8),V(8),E(8),C2B2(8,8)
      REAL WKAREA(8),MA11(8,8),MA12(8,8),F2B2(8,8)
      REAL SIGMA(8,8),F2INV(8,8),K00(8,8),K11(8,8)
      REAL BETA(8),SLOW2(8),AFIF(8,8),F2IF1(8,8),F2INV(8,8)
      COMPLEX ALFA(8),LAMBDA(8),LAM(8),W(8),W1(8),ZQ(8,8)
      REAL KAY(8,8),ACL(8,8)
      REAL C2KAY(8,8),C1C2K(8,8),ACLINV(8,8)
      REAL ACLO(8,8),A12KAY(8,8)
      REAL AY(8,8),BEE(8,8),CEE(8,8),B2K1(8,8),B2K0(8,8)
      REAL B2K0F1(8,8),B2K0F2(8,8),BKF2(8,8),BK1(8,8)
      REAL BKF1(8,8),THRTWO(8,8),THRTHR(8,8),GB2K0(8,8)
      REAL IDEN(8,8),F2S(8,8)
      PRINT*,'STANDARD AFTI MODEL ?(1=YES) :'
      READ*,XX
      DO 99 I=1,2
      PRINT*,' '
99    CONTINUE
      PRINT*,' WELCOME TO PORTER'S METHOD FOR IRREGULAR PLANTS .'
      PRINT*,' VERSION 1.2    CURRENTLY THIS PROGAM CAN HANDLE UP '
      PRINT*,' TO 8 TOTAL STATES , WITH REAL OR COMPLEX DESIRED '
      PRINT*,' SLOW EIGENVALUES .'
      PRINT*,' '
      PRINT*,'ENTER THE DIMENSION OF THE SYSTEM :'
      READ*,N
      IF (N.LE.7) GOTO 100
      PRINT*,'THE DIMENSIONS MUST BE CHANGED TO HANDLE THAT'
      GOTO 9999
100   PRINT*,'ENTER THE NUMBER OF ACTIVE INPUTS (SIZE OF B2) :'
      READ*,L
      NM=N-L
      IF (XX.EQ.1) GOTO 8045
      PRINT*,'ENTER A11 -'
      CALL READM(NM,NM,A11)
      PRINT*,' '
      PRINT*,'ENTER A12 -'
      CALL READM(NM,L,A12)
      PRINT*,' '
      PRINT*,'ENTER A21 -'
      CALL READM(L,NM,A21)
      PRINT*,' '
```

```
560-        PRINT*,'ENTER A22 -'
570-        CALL READM(L,L,A22)
580-        PRINT*,' '
590-        PRINT*,'B1 IS A ZERO MATRIX OF DIMENSION ',NM,' BY ',L
600-        DO 200 I=1,NM
610-        DO 210 J=1,L
620-        B1(I,J)=0
630-210     CONTINUE
640-200     CONTINUE
650-        PRINT*,' '
660-        PRINT*,'ENTER B2 -'
670-        CALL READM(L,L,B2)
680-        GOTO 8592
690-8045    CONTINUE
700-        CALL AFTI(A11,A12,A21,A22,B1,B2)
710-        GOTO 8592
720-7649    CONTINUE
730-        PRINT*,' '
740-        PRINT*,'NEW OUTPUT MATRICES ?(1=YES) :'
750-        READ*,XS
760-        IF (XS.NE.1) GOTO 380
770-8592    CONTINUE
780-        PRINT*,' '
790-        PRINT*,'ENTER C1 -'
800-        CALL READM(L,NM,C1)
810-        PRINT*,' '
820-        PRINT*,'ENTER C2 -'
830-        CALL READM(L,L,C2)
840-        PRINT*,' '
850-        PRINT*,'THIS METHOD IS FOR IRREGULAR PLANTS, THAT IS, FOR'
860-        PRINT*,'C2B2 NOT OF FULL RANK'
870-        CALL VMULFF(C2,B2,L,L,L,8,8,C2B2,8,IER)
880-        D1=1.
890-        CALL LINV3F(C2B2,B,4,L,8,D1,D2,WKAREA,IER)
900-        DET=D1*2**D2
910-        IF (DET.EQ.0) GOTO 280
920-        PRINT*,'THIS PLANT IS REGULAR - PROGRAM TERMINATED'
930-        GOTO 9999
940-280     PRINT*,'THE PLANT IS IRREGULAR'
950-        PRINT*,' '
960-380     CONTINUE
970-610     PRINT*,' '
980-        CALL NULLSP(A11,A12,ACL,NM,L,KAY)
990-        DO 972 I=1,NM
1000-       DO 971 J=1,L
```

```
      A12(I,J)=-A12(I,J)
971   CONTINUE
972   CONTINUE
      CALL VMULFF(C2,KAY,L,L,NM,8,8,C2KAY,8,IER)
      CALL MSUB(C1,C2KAY,C1C2K,L,NM)
      CALL VMULFF(A12,KAY,NM,L,NM,8,8,A12KAY,8,IER)
      CALL MSUB(A12KAY,A11,ACLO,NM,NM)
      CALL LINV1F(ACLO,NM,8,ACLINV,8,WKAREA,IER)
      CALL VMULFF(C1C2K,ACLINV,L,NM,NM,8,8,M,8,IER)
      PRINT*,' '
      PRINT*,'THE M MATRIX IS -'
      CALL PRINTM(L,NM,M)
      CALL VMULFF(M,A11,L,NM,NM,8,8,MA11,8,IER)
      CALL MADD(C1,MA11,L,NM,F1)
      CALL VMULFF(M,A12,L,NM,L,8,8,MA12,8,IER)
      CALL MADD(C2,MA12,L,L,F2)
      CALL MADD(C2,MA12,L,L,F2S)
      CALL VMULFF(F2,B2,L,L,L,8,8,F2B2,8,IER)
      PRINT*,' '
      PRINT*,'THE MATRIX F1 IS -'
      CALL PRINTM(L,NM,F1)
      PRINT*,' '
      PRINT*,'THE MATRIX F2 IS -'
      CALL PRINTM(L,L,F2)
      PRINT*,' '
      PRINT*,'THE MATRIX F2*B2 IS -'
      CALL PRINTM(L,L,F2B2)
      D1=1
      DO 365 I=1,L
      DO 364 J=1,L
      F2B2D(I,J)=F2B2(I,J)
364   CONTINUE
365   CONTINUE
      CALL LINV3F(F2B2D,B,4,L,8,D1,D2,WKAREA,IER)
      DET=D1*2**D2
      IF (DET.NE.0) GOTO 370
      PRINT*,'THE MATRIX F2*B2 IS NOT FULL RANK - TRY AGAIN'
      GOTO 7649
370   CONTINUE
      PRINT*,' '
      PRINT*,'NOW VALUES OF SIGMA MUST BE CHOSEN. HOW DO YOU WANT'
      PRINT*,'TO INPUT THESE (0=DIAGONAL, 1=FULL) :'
      READ*,JJ
      IF (JJ.EQ.1) GOTO 390
      DO 393 I=1,L
```

```
      DO 394 J=1,L
      SIGMA(I,J)=0
394   CONTINUE
393   CONTINUE
      PRINT*,' '
      PRINT*,'ENTER THE ',L,' VALUES OF SIGMA (MUST BE POSITIVE) :'
      DO 400 I=1,L
      READ*,SIGMA(I,I)
400   CONTINUE
      GOTO 410
390   CONTINUE
      PRINT*,' '
      PRINT*,'ENTER SIGMA -'
      CALL READM(L,L,SIGMA)
410   CONTINUE
      CALL LINV1F(F2B2,L,8,FBINV,8,WKAREA,IER)
      CALL VMULFF(FBINV,SIGMA,L,L,L,8,8,K0,8,IER)
      PRINT*,' '
      PRINT*,'THE MATRIX K0 IS -'
      CALL PRINTM(L,L,K0)
      DO 460 I=1,L
      DO 470 J=1,L
      K00(I,J)=-K0(I,J)
470   CONTINUE
460   CONTINUE
      PRINT*,' '
      PRINT*,'THE MATRIX K1 MUST NOW BE INPUT. HOW DO YOU WISH'
      PRINT*,'TO DO THIS (0=SCRATCH , 1=MULTIPLE OF K0) :'
      READ*,G
      IF (G.EQ.1) GOTO 480
520   CONTINUE
      PRINT*,' '
      PRINT*,'ENTER K1 -'
      CALL READM(L,L,K1)
      DO 500 I=1,L
      DO 490 J=1,L
      K11(I,J)=K1(I,J)
490   CONTINUE
500   CONTINUE
      GOTO 510
480   CONTINUE
      PRINT*,' '
      PRINT*,'WHAT MULTIPLE OF K0 IS K1 :'
      READ*,Q
      DO 521 I=1,L
```

```
      DO 502 J=1,L
      K1(I,J)=Q*K0(I,J)
      K11(I,J)=K1(I,J)
502   CONTINUE
501   CONTINUE
510   CONTINUE
      CALL EIGZF(K11,8,K00,8,L,0,ALFA,BETA,Z0,8,WKAREA,IER)
      DO 530 I=1,L
      YY=BETA(I)
      IF (YY.EQ.0) GOTO 540
      LAMBDA(I)=ALFA(I)/BETA(I)
      GOTO 530
540   LAMBDA(I)=9.E+99
530   CONTINUE
      PRINT*,' '
      PRINT*,'THE FIRST SET OF SLOW EIGENVALUES ARE :'
      PRINT*,' '
      DO 550 I=1,L
      PRINT*,'LAMBDA(',I,') = ',LAMBDA(I)
      PRINT*,' '
550   CONTINUE
      CALL LINV1F(F2S,L,8,F2INV,8,WKAREA,IER)
      CALL VMULFF(F2INV,F1,L,L,NM,8,8,F2IF1,8,IER)
      CALL VMULFF(A12,F2IF1,NM,L,NM,8,8,A12FF,8,IER)
      DO 560 I=1,NM
      DO 570 J=1,NM
      AFIF(I,J)=-A12FF(I,J)
570   CONTINUE
560   CONTINUE
      CALL MADD(A11,AFIF,NM,NM,SLOW2)
      CALL EIGRF(SLOW2,NM,8,0,W,Z0,8,WKAREA,IER)
      PRINT*,' '
      PRINT*,'THE SECOND SET OF SLOW EIGENVALUES ARE :'
      PRINT*,' '
      DO 580 I=1,NM
      J=I+L
      PRINT*,'LAMBDA(',J,') = ',W(I)
      PRINT*,' '
580   CONTINUE
      CALL EIGRF(SIGMA,L,8,0,W1,Z0,8,WKAREA,IER)
      PRINT*,'THE FAST EIGENVALUES ARE (MUST BE MULTIPLIED BY G) :'
      PRINT*,' '
      DO 590 I=1,L
      J=NM+L+I
      LAM(I)=-W1(I)
```

```
      PRINT*,'LAMBDA(',J,') = ',LAM(I)
      PRINT*,' '
690   CONTINUE
      PRINT*,'FORM CLOSED LOOP MATRICES (1), CHANGE OUTPUTS/EIGENVALUES'
      PRINT*,' (2), OR STOP (0) :'
      READ*,X3
      IF (X3.EQ.2) GOTO 7649
      IF (X3.EQ.0) GOTO 9999
3999  PRINT*,' '
      PRINT*,'PRINT C-L MATRICES (1=YES) :'
      READ*,X25
      PRINT*,' '
      PRINT*,'ENTER THE GAIN CONSTANT , G :'
      READ*,G
      DO 4000 I=1,N
      DO 3995 J=1,L
      AY(I,J)=0.
3995  CONTINUE
4000  CONTINUE
      CALL VMULFF(B2,K1,L,L,L,8,8,B2K1,8,IER)
      CALL VMULFF(B2,K0,L,L,L,8,8,B2K0,8,IER)
      CALL VMULFF(B2K0,F1,L,L,NM,8,8,B2K0F1,8,IER)
      CALL VMULFF(B2K0,F2,L,L,L,8,8,B2K0F2,8,IER)
      DO 4100 I=1,L
      DO 4090 J=1,L
      BKF2(I,J)=G*B2K0F2(I,J)
      BK1(I,J)=G*B2K1(I,J)
4090  CONTINUE
      DO 4095 KA=1,NM
      BKF1(I,KA)=G*B2K0F1(I,KA)
4095  CONTINUE
4100  CONTINUE
      CALL MSUB(A21,BKF1,THRTWO,L,NM)
      CALL MSUB(A22,BKF2,THRTHR,L,L)
      DO 4200 I=1,L
      DO 4190 J=1,L
      KA=I+N
      LA=J+N
      AY(KA,J)=BK1(I,J)
      AY(I,LA)=-F2(I,J)
      AY(KA,LA)=THRTHR(I,J)
4190  CONTINUE
      DO 4195 J=1,NM
      LB=J+L
      AY(I,LB)=-F1(I,J)
```

```
      AY(KA,LB)=THRTWO(I,J)
4195  CONTINUE
4200  CONTINUE
      DO 4300 I=1,NM
      DO 4290 J=1,L
      LA=I+L
      KA=J+N
      AY(LA,KA)=A12(I,J)
4290  CONTINUE
      DO 4295 J=1,NM
      KB=J+L
      AY(LA,KB)=A11(I,J)
4295  CONTINUE
4300  CONTINUE
      NL=N+L
      IF (X25.NE.1) GOTO 4310
      PRINT*,' '
      PRINT*,'THE CLOSED LOOP AY MATRIX IS -'
      CALL PRINTM(NL,NL,AY)
      DO 4305 I=1,NL
      DO 4304 J=1,NL
      WRITE (3,10) AY(I,J)
4304  CONTINUE
4305  CONTINUE
10    FORMAT(F20.7)
4310  CONTINUE
      DO 4400 I=1,NM
      DO 4395 J=1,L
      KA=I+L
      BEE(KA,J)=0.
4395  CONTINUE
4400  CONTINUE
      DO 4500 I=1,L
      DO 4495 J=1,L
      GB2K0(I,J)=G*B2K0(I,J)
      KA=I+N
      IDEN(I,J)=0.
      IDEN(I,I)=1.
      BEE(I,J)=IDEN(I,J)
      BEE(KA,J)=GB2K0(I,J)
4495  CONTINUE
4500  CONTINUE
      IF (X25.NE.1) GOTO 4510
      PRINT*,' '
      PRINT*,'THE CLOSED LOOP BEE MATRIX IS -'
```

```
3260-      CALL PRINTM(NL,L,BEE)
3270-      DO 4505 I=1,NL
3280-      DO 4504 J=1,L
3290-      WRITE (3,10) BEE(I,J)
3300-4504  CONTINUE
3310-4505  CONTINUE
3320-4510  CONTINUE
3330-      DO 4600 I=1,L
3340-      DO 4590 J=1,L
3350-      KA=J+M
3360-      CEE(I,J)=0.
3370-      CEE(I,KA)=C2(I,J)
3380-4590  CONTINUE
3390-      DO 4595 J=1,MM
3400-      KB=J+L
3410-      CEE(I,KB)=C1(I,J)
3420-4595  CONTINUE
3430-4600  CONTINUE
3440-      PRINT*,' '
3450-      IF (X25.NE.1) GOTO 4610
3460-      PRINT*,'THE CLOSED LOOP CEE MATRIX IS -'
3470-      CALL PRINTM(L,NL,CEE)
3480-      DO 4605 I=1,L
3490-      DO 4604 J=1,NL
3500-      WRITE(3,10) CEE(I,J)
3510-4604  CONTINUE
3520-4605  CONTINUE
3530-4610  CONTINUE
3540-      PRINT*,' '
3550-      PRINT*,'THE OVERALL CLOSED LOOP EIGENVALUES ARE :'
3560-      PRINT*,' '
3570-      CALL EIGRF(AY,NL,8,0,W1,Z0,8,WKAREA,IER)
3580-      DO 4700 I=1,NL
3590-      PRINT*,' EIGENVALUE (',I,') = ',W1(I)
3600-4700  CONTINUE
3610-      PRINT*,' '
3620-      PRINT*,' '
3630-      PRINT*,' '
3640-      PRINT*,'CHANGE OUTPUTS/EIGENVALUES (1), CHANGE ONLY GAIN (2),'
3650-      PRINT*,'OR STOP (3) :'
3660-      READ*,X10
3670-      IF (X10.EQ.1) GOTO 7649
3680-      IF (X10.EQ.2) GOTO 3999
3690-      PRINT*,' '
3700-      PRINT*,'ALL INFORMATION IN PORTER HAS BEEN SAVED IN LOCAL'
```

```
PRINT*,'FILE - CDATA'
9999  STOP
      END
C
C
C
      SUBROUTINE MADD(A,B,L,N,C)
      DIMENSION A(8,8),B(8,8),C(8,8)
      DO 1000 I=1,L
      DO 1010 J=1,N
      C(I,J)=A(I,J)+B(I,J)
1010  CONTINUE
1000  CONTINUE
      RETURN
      END
C
      SUBROUTINE READM(L,N,A)
      REAL A(8,8)
      PRINT*,' '
      DO 1500 I=1,L
      PRINT*,' ROW ',I,' :'
      READ*,(A(I,J),J=1,N)
1500  CONTINUE
      RETURN
      END
C
      SUBROUTINE PRINTM(L,N,A)
      REAL A(8,8)
      PRINT*,' '
      DO 1700 I=1,L
      PRINT 738,(A(I,J),J=1,N)
1700  CONTINUE
738   FORMAT(1X,8(F12.4,3X))
      RETURN
      END
C
C
      SUBROUTINE MSUB(A,B,C,L,N)
      REAL A(8,8),B(8,8),C(8,8)
      DO 347 I=1,L
      DO 346 J=1,N
      C(I,J)=A(I,J)-B(I,J)
346   CONTINUE
347   CONTINUE
      RETURN
```

```
      END
C
C
      SUBROUTINE NULLSP(A,B,ACL,NA,MB,K)
      DIMENSION A(8,8),B(8,8),S(20,20)
      DIMENSION U(20,20),SING(20),WORK(40)
      REAL EIGN(8,8),OMEGA(8,8),MU(8,8),NUINU(8,8)
      REAL K(8,8),BK(8,8),ACL(8,8)
      COMPLEX W(8),Z(8,8)
      REAL MUR,MUI
      REAL MUL(20)
      REAL EIGEN(20,20)
      COMMON /NULL/ EIGEN,KK
      NDIMA=8 $ NDIMB=8 $NDIMS=20 $NDIMU=20
22    CONTINUE
      KK=0
100   DO 2 I=1,NDIMU
      DO 1 J=1,NDIMU
1     U(I,J)=0.
2     U(I,I)=1.
      DO 25 I=1,NA
      DO 24 J=1,MB
      B(I,J)=-B(I,J)
24    CONTINUE
25    CONTINUE
      PRINT*,"HOW MANY PAIRS OF COMPLEX SLOW EIGENVALUES :"
      READ*,NCOMP
      NX=NA-NCOMP
      DO 500 MCOUNT=1,NX
      PRINT*,"ENTER REAL AND IMAG PARTS OF SLOW EIGENVALUE ",MCOUNT," :"
      READ *,MUR,MUI
      NS=NA
      MS=NA+MB
      IF (ABS(MUI).LE..0000001) GO TO 10
C     DESIRED EIGENVALUE IS COMPLEX
      NS=2*NS
      MS=2*MS
10    CONTINUE
      CALL SETUP(A,B,S,NA,MB,NS,MS,MUR,MUI,NDIMA,NDIMB,NDIMS)
      NOTU=-1
      CALL LSVDF(S,NDIMS,NS,MS,U,NS,NOTU,SING,WORK,IER)
      PRINT *," IER =",IER
      CALL KERNEL(MS,SING,S,NDIMS,MUI)
500   CONTINUE
      PRINT*,"THE ENTIRE EIGENVECTOR MATRIX IS -"
```

```
      DO 333 LMN=1,MS
      PRINT 376,(EIGEN(LMN,MNL),MNL=1,KK)
333   CONTINUE
376   FORMAT(1X,10(F8.4,8X))
      NAD=2*NA
      PRINT*,' '
      PRINT*,'NOW ',NAD,' NUMBERS MUST BE INPUT, DENOTING THE CONSTANT'
      PRINT*,'MULTIPLIER BEFORE EACH EIGENVECTOR :'
      DO 2000 I=1,NAD
      READ*,MUL(I)
      DO 1999 J=1,MS
      EIGEN(J,I)=MUL(I)*EIGEN(J,I)
1999  CONTINUE
2000  CONTINUE
      DO 2010 I=1,NA
      DO 2009 J=1,MS
      EIGN(J,I)=0
2009  CONTINUE
2010  CONTINUE
      L1=-1
      L2=0
      DO 2020 I=1,NA
      L1=L1+2
      L2=L2+2
      DO 2019 J=1,MS
      DO 2018 KO=L1,L2
      EIGN(J,I)=EIGN(J,I)+EIGEN(J,KO)
2018  CONTINUE
2019  CONTINUE
2020  CONTINUE
      DO 2030 I=1,NA
      DO 2029 J=1,NA
      NU(I,J)=EIGN(I,J)
2029  CONTINUE
2030  CONTINUE
      NNN=NA+1
      DO 2040 I=NNN,MS
      DO 2039 J=1,NA
      LD=I-NA
      OMEGA(LD,J)=EIGN(I,J)
2039  CONTINUE
2040  CONTINUE
      PRINT*,' '
      PRINT*,'THE NU MATRIX IS -'
      CALL PRINTM(NA,NA,NU)
```

```
      PRINT*,' '
      PRINT*,'THE OMEGA MATRIX IS -'
      CALL PRINTM(MB,NA,OMEGA)
      CALL LINV1F(NU,NA,8,NUINV,8,UKAREA,IER)
      PRINT*,' '
      PRINT*,'THE NU INVERSE MATRIX IS -'
      CALL PRINTM(NA,NA,NUINV)
      CALL VMULFF(OMEGA,NUINV,MB,NA,NA,8,8,K,8,IER)
      PRINT*,' '
      PRINT*,'THE K MATRIX IS -'
      CALL PRINTM(MB,NA,K)
      CALL VMULFF(B,K,NA,MB,NA,8,8,BK,8,IER)
      CALL MADD(A,BK,NA,NA,ACL)
      PRINT*,' '
      PRINT*,'THE CLOSED LOOP A MATRIX IS -'
      CALL PRINTM(NA,NA,ACL)
      CALL EIGRF(ACL,NA,8,0,W,Z,8,WKAREA,IER)
      PRINT*,' '
      PRINT*,'THE CLOSED LOOP SLOW EIGENVALUES ARE -'
      DO 2100 I=1,NA
      PRINT*,W(I)
2100  CONTINUE
50    RETURN
      END
C
      SUBROUTINE SETUP(A,B,S,NA,MB,NS,MS,MUR,MUI,NDIMA,NDIMB,NDIMS)
      DIMENSION A(NDIMA,NA),B(NDIMB,MB),S(NDIMS,MS)
      REAL MUR,MUI
C     SETUP THE FOLLOWING 'S' MATRIX:
C
C     IF THE DESIRED EIGENVLUE 'MU' IS REAL, THEN:
C
C                   S = (A-MU*I,B)
C
C     IF THE DESIRED EIGENVALUE IS COMPLEX, THEN:
C
C                          (A-MUR*I, B,  MUI*I , 0)
C                   S =
C                          ( -MUI*I, 0, A-MUR*I , B)
C
      DO 40 I=1,NA
      DO 20 J=1,NA
      S(I,J)=A(I,J)
20    CONTINUE
      DO 30 J=1,MB
```

```
      JPNA=J+NA
      B(I,JPNA)=B(I,J)
30    CONTINUE
      S(I,I)=S(I,I)-MUR
40    CONTINUE
      IF (ABS(MUI).LE..0000001) GO TO 200
C     EIGENVALUE MUST BE COMPLEX, THEREFORE FORM EXPANDED S
      NAPMB=NA+MB
      DO 70 I=1,NA
      DO 60 J=1,NAPMB
      IPNA=I+NA
      JNAMB=J+NA+MB
      S(IPNA,J)=0.
      S(I,JNAMB)=0.
      S(IPNA,JNAMB)=S(I,J)
60    CONTINUE
      S(IPNA,I)=-MUI
      S(I,I+NA+MB)=MUI
70    CONTINUE
200   CONTINUE
      RETURN
      END
C
      SUBROUTINE KERNEL(MS,SING,S,NDIMS,MUI)
      DIMENSION SING(MS),S(NDIMS,MS)
      REAL EIGEN (20,20)
      COMMON /NULL/ EIGEN,KK
      REAL MUI
      PRINT *,' VECTORS SPANNING THE NULL SPACE ARE:'
      EIGMAX=0
      DO 50 I=2,MS
         IF (SING(I)/SING(1) .GT. 1.E-6)  GO TO 30
      EIGMAX=EIGMAX+1
      IF (EIGMAX.GT.2) GOTO 30
C     ZERO SINGULAR VALUE, THEREFORE VECTOR LIES IN NULL SPACE
          PRINT*,'  SINGULAR VALUE=',SING(I),'   CORRESPONDING VECTOR:'
      IF (ABS(MUI).GT..0000001) GOTO 300
          PRINT 1000,(S(J,I),J=1,MS)
      KK=KK+1
      DO 200 J=1,MS
      EIGEN(J,KK)=S(J,I)
200   CONTINUE
      GOTO 30
300   CONTINUE
C     COMPLEX VECTOR
```

```
              PRINT 1020
              MS2=MS/2
              DO 20 J=1,MS2
                SMAG=SQRT(S(J,I)**2+S(J+MS2,I)**2)
                SPHASE=ATAN2(S(J+MS2,I),S(J,I))
                PRINT 1000,S(J,I),S(J+MS2,I),SMAG,SPHASE*57.296
20              CONTINUE
         KK=KK+1
         DO 400 J=1,MS2
         EIGEN(J,KK)=S(J,I)
400      CONTINUE
         DO 500 J=1,MS2
         EIGEN(J,KK+2)=S(J+MS2,I)
500      CONTINUE
30       CONTINUE
50       CONTINUE
         IF(ABS(MUI).GT..0000001) KK=KK+2
         RETURN
1000     FORMAT(1X,6F10.4/6X,6F10.4/6X,6F10.4/6X,6F10.4)
1020     FORMAT(8X,'COMPLEX VECTOR'/4X,'REAL',7X,'IMAG',8X,'MAG',8X,'PHASE
        1 -DEG')
         END
C
         SUBROUTINE AFTI(A,B,C,D,E,F)
         REAL A(8,8),B(8,8),C(8,8),D(8,8),E(8,8),F(8,8)
         A(1,1)=-.01505 $A(1,2)=-12.823 $A(1,3)=0. $A(1,4)=-32.174
         A(2,1)=-.000146 $A(2,2)=-1.3411 $A(2,3)=.99335 $A(2,4)=0.
         A(3,1)=.00018 $A(3,2)=43.233 $A(3,3)=-.86939 $A(3,4)=0.
         A(4,1)=0. $A(4,2)=0. $A(4,3)=1. $A(4,4)=0.
         B(1,1)=-2.5157 $B(1,2)=-13.136
         B(2,1)=-.16897 $B(2,2)=-.25183
         B(3,1)=-17.251 $B(3,2)=-1.5766
         B(4,1)=0. $B(4,2)=0.
         DO 5000 I=1,4
         DO 5001 J=1,4
         C(I,J)=0.
         E(I,J)=0.
5001     CONTINUE
5000     CONTINUE
         D(1,1)=-20. $D(1,2)=0.
         D(2,1)=0. $D(2,2)=-20.
         F(1,1)=20. $F(1,2)=0.
         F(2,1)=0. $F(2,2)=20.
         PRINT*,' '
         PRINT*,'THE A11 MATRIX IS - '
```

```
      CALL PRINTM(4,4,A)
      PRINT*,' '
      PRINT*,'THE A12 MATRIX IS -'
      CALL PRINTM(4,2,B)
      PRINT*,' '
      PRINT*,'THE A21 MATRIX IS -'
      CALL PRINTM(2,4,C)
      PRINT*,' '
      PRINT*,'THE A22 MATRIX IS -'
      CALL PRINTM(2,2,D)
      PRINT*,' '
      PRINT*,'THE B1 MATRIX IS A 4 BY 2 ZERO MATRIX'
      PRINT*,' '
      PRINT*,'THE B2 MATRIX IS -'
      CALL PRINTM(2,2,F)
      RETURN
      END
```

```
STANDARD AFTI MODEL ?(1=YES) 1
    450008   CM STORAGE USED
       2.138 CP SECONDS COMPILATION TIME  0

 WELCOME TO PORTER'S METHOD FOR IRREGULAR PLANTS ,
 VERSION 1.2    CURRENTLY THIS PROGAM CAN HANDLE UP
 TO 8 TOTAL STATES , WITH REAL OR COMPLEX DESIRED
 SLOW EIGENVALUES .

ENTER THE DIMENSION OF THE SYSTEM : 5
ENTER THE NUMBER OF ACTIVE INPUTS (SIZE OF B2) : 2
ENTER A11 -

 ROW 1 : 0,1,0
 ROW 2 : 0,-.86939,43.233
 ROW 3 : 0,.99335,-1.3411

ENTER A12 -

 ROW 1 : 0,0
 ROW 2 : -17.251,-1.5766
 ROW 3 : -.16897,-.25183

ENTER A21 -

 ROW 1 : 0,0,0
 ROW 2 : 0,0,0

ENTER A22 -

 ROW 1 : -20,0
 ROW 2 : 0,-20

B1 IS A ZERO MATRIX OF DIMENSION 3 BY 2

ENTER B2 -

 ROW 1 : 20,0
 ROW 2 : 0,20

ENTER C1 -

 ROW 1 : 1,0,0
 ROW 2 : 1,0,-1

ENTER C2 -

 ROW 1 : 0,0
 ROW 2 : 0,0
```

```
THIS METHOD IS FOR IRREGULAR PLANTS, THAT IS, FOR
CB82 NOT OF FULL RANK
*** TERMINAL ERROR          (IER = 129) FROM IMSL ROUTINE LUD
ATF
*** TERMINAL ERROR          (IER = 130) FROM IMSL ROUTINE LIN
V3F
THE PLANT IS IRREGULAR

HOW MANY PAIRS OF COMPLEX SLOW EIGENVALUES : 1
ENTER REAL AND IMAG PARTS OF SLOW EIGENVALUE 1 : -4,3
 IER =0
 VECTORS SPANNING THE NULL SPACE ARE:
  SINGULAR VALUE=0.   CORRESPONDING VECTOR:
       COMPLEX VECTOR
   REAL        IMAG         MAG       PHASE  -DEG
   -.1569      .0000      .1569  180.0007
    .6278     -.4708      .7847  -36.8700
   -.1867     -.0626      .1969 -161.4694
    .4409      .3458      .5603   38.1068
   -.0556      .0620      .0832  131.8732
  SINGULAR VALUE=0.   CORRESPONDING VECTOR:
       COMPLEX VECTOR
   REAL        IMAG         MAG       PHASE  -DEG
   0.0000      .0295      .0295   90.0003
   -.0886     -.1181      .1476 -126.8704
    .0237      .0853      .0885   74.4781
    .0650     -.2064      .2164  -72.5183
   -.9605     -.0141      .9606 -179.1611
ENTER REAL AND IMAG PARTS OF SLOW EIGENVALUE 2 : -3,0
 IER =0
 VECTORS SPANNING THE NULL SPACE ARE:
  SINGULAR VALUE=0.   CORRESPONDING VECTOR:
    -.1569     .4706     -.3046      .7367     -.3442
  SINGULAR VALUE=0.   CORRESPONDING VECTOR:
    0.0000    0.0000     -.1756      .3562      .9178
THE ENTIRE EIGENVECTOR MATRIX IS -
-.1569  0.0000   .0000   .0295  -.1569  0.0000
 .6278  -.0886  -.4708  -.1181   .4706  0.0000
-.1867   .0237  -.0626   .0853  -.3046  -.1756
 .4409   .0650   .3458  -.2064   .7367   .3562
-.0556  -.9605   .0620  -.0141  -.3442   .9178

NOW 6 NUMBERS MUST BE INPUT, DENOTING THE CONSTANT
MULTIPLIER BEFORE EACH EIGENVECTOR : 1,1,1,1,1,0
```

```
THE NU MATRIX IS -

     -.1569           .0295          -.1569
      .6392          -.5889           .4706
     -.1630           .0287          -.3046

THE OMEGA MATRIX IS -

      .5059           .1393           .7367
    -1.0160           .0479          -.3448

THE NU INVERSE MATRIX IS -

   -15.6870          -.5048          7.2994
    -8.1401         -2.0676           .9977
     7.7876           .1159         -7.1148

THE K MATRIX IS -

    -3.3334          -.4581         -1.4095
    12.8677           .3739         -4.9195

THE CLOSED LOOP A MATRIX IS -

     0.0000          1.0000          0.0000
   -37.2164         -8.1819         11.1611
     2.6772          1.0101         -2.8181

THE CLOSED LOOP SLOW EIGENVALUES ARE -
(-3.999999999998,2.999999999999)
(-3.999999999998,-2.999999999999)
(-2.999999999999,0.)

THE M MATRIX IS -

      .1571           .0376           .1488
      .3663           .0019          -.3474

THE MATRIX F1 IS -

     1.0000           .2723          1.4249
     1.0000           .0196          -.4529

THE MATRIX F2 IS -

     -.6734          -.0967
      .0263           .0845

THE MATRIX F2*B2 IS -

   -13.4671         -1.9343
      .5258          1.6905

NOW VALUES OF SIGMA MUST BE CHOSEN. HOW DO YOU WANT
TO INPUT THESE (0=DIAGONAL, 1=FULL) : 0

ENTER THE 2 VALUES OF SIGMA (MUST BE POSITIVE) : 1,1

THE MATRIX K0 IS -

     -.0777          -.0889
      .0242           .6192

THE MATRIX K1 MUST NOW BE INPUT. HOW DO YOU WISH
TO DO THIS (0=SCRATCH , 1=MULTIPLE OF K0) : 1

WHAT MULTIPLE OF K0 IS K1 : 2

THE FIRST SET OF SLOW EIGENVALUES ARE :

LAMBDA(1) = (-2.,0.)

LAMBDA(2) = (-2.,0.)


THE SECOND SET OF SLOW EIGENVALUES ARE :

LAMBDA(3) = (-3.999999999998,2.999999999999)

LAMBDA(4) = (-3.999999999998,-2.999999999999)

LAMBDA(5) = (-2.999999999999,0.)

THE FAST EIGENVALUES ARE (MUST BE MULTIPLIED BY G) :

LAMBDA(6) = (-1.,0.)

LAMBDA(7) = (-1.,0.)

FORM CLOSED LOOP MATRICES (1), CHANGE OUTPUTS/EIGENVALUES
 (2), OR STOP (0) : 1

PRINT C-L MATRICES (1=YES) : 2
```

```
ENTER THE GAIN CONSTANT , G : 500

THE OVERALL CLOSED LOOP EIGENVALUES ARE :

 EIGENVALUE (1) = (-511.115664743,0.)
 EIGENVALUE (2) = (-510.5569229567,0.)
 EIGENVALUE (3) = (-3.619649883994,2.853980420868)
 EIGENVALUE (4) = (-3.619649883994,-2.853980420868)
 EIGENVALUE (5) = (-3.268768969414,0.)
 EIGENVALUE (6) = (-2.014916766373,.174182069864)
 EIGENVALUE (7) = (-2.014916766373,-.174182069864)


CHANGE OUTPUTS/EIGENVALUES (1), CHANGE ONLY GAIN (2),
OR STOP (3) : 2

PRINT C-L MATRICES (1=YES) : 1

ENTER THE GAIN CONSTANT , G : 1000

THE CLOSED LOOP AY MATRIX IS -

      0.0000       0.0000      -1.0000      -.2783     -1.4249       .6734       .0967
      0.0000       0.0000      -1.0000      -.0196       .4529      -.0263      -.0845
      0.0000       0.0000       0.0000      1.0000      0.0000      0.0000      0.0000
      0.0000       0.0000       0.0000      -.8694     43.2330    -17.2510     -1.5766
      0.0000       0.0000       0.0000       .9934     -1.3411      -.1690      -.2518
  -3109.1183   -3557.5872    3333.3527    458.0607   1409.5330  -1020.0000      -.0000
    967.1247   24768.2842  -12867.7045   -373.9257   4919.4574      0.0000  -1020.0000

THE CLOSED LOOP BEE MATRIX IS -

      1.0000       0.0000
      0.0000       1.0000
      0.0000       0.0000
      0.0000       0.0000
      0.0000       0.0000
  -1554.5591   -1778.7936
    483.5623   12384.1421

THE CLOSED LOOP CEE MATRIX IS -

      0.0000       0.0000       1.0000      0.0000      0.0000      0.0000      0.0000
      0.0000       0.0000       1.0000      0.0000     -1.0000      0.0000      0.0000
```

VITA

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The theory of high-gain error-actuated feedback control, developed by Porter and Bradshaw, was applied to the design of various longitudinal decoupling flight control systems for an advanced aircraft. This aircraft incorporates both pitching moment (elevators) and direct lift (flaps) control devices, permitting the application of multivariable control theory. The controllers developed in this study utilized output feedback with

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20. proportional plus integral control to produce desirable closed-loop response with minimal interaction between outputs. Because of the structure of the system, measurement variables different from the outputs are necessary to apply this method. This report describes how entire eigenstructure assignment can be used to determine appropriate measurement equations by assigning their corresponding transmission zeros. A singular value decomposition was used to choose the eigenvectors from their permissible subspaces. The results show that these modes appear in the output response, even for very high gain. Therefore, selection of good eigenvalues/eigenvectors was shown to be crucial to the successful application of this theory. In addition, this report examined the effect of varying the other design parameters on the closed-loop system response.

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